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• Review •

A Scoping Review of the Application of Health Portraits in Cognitive Impairment Care

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Abstract Porous media oil-water two-phase fluid-solid coupling is critical for oil-gas exploitation efficiency and environmental protection, but traditional mesh-based methods (FDM, FEM, FVM) suffer from mesh generation difficulties, poor dynamic adaptability, and numerical dissipation. The Generalized Finite Difference Method (GFDM), a promising meshless approach, offers a solution. This review synthesizes research progress on upwind GFDM in this coupling problem. It first sorts the evolution of porous media fluid-solid coupling theory (from Terzaghi's to multi-physics extensions) and summarizes mesh-based/meshless methods, focusing on GFDM's development and cross-field applications. It then analyzes how upwind-GFDM integration enhances convection-dominated flow stability/accuracy, addressing traditional meshless upwind challenges. Key gaps are identified: limited GFDM use in fractured porous media, unoptimized node layout/influence domains, and insufficient AI integration. Finally, the review concludes upwind GFDM provides a novel technical pathway for complex coupling problems, laying a foundation for meshless porous media simulators. It outlines future directions (fractured reservoir expansion, parameter optimization, AI fusion) to improve exploitation efficiency and environmental protection.

Keywords Closed-loop optimization; History matching; Production optimization; Optimization algorithm

1 Introduction

Porous media are widely present in nature and engineering practices, such as underground oil reservoirs, soil, and rocks. The oil-water two-phase flow behavior in underground oil reservoirs directly affects the efficiency of oil and gas exploitation and the risk of environmental pollution. By studying the fluid-solid coupling problem of oil-water two-phase flow in porous media, scientific basis can be provided for the rational exploitation of underground oil reservoirs and environmental protection work such as the prevention of groundwater pollution.

Scholars have conducted research on the fluid-solid coupling problem in porous media and achieved a series of remarkable results in aspects such as fluid-solid coupling theory, numerical solution technology, and engineering applications. Traditional mesh-based numerical solution methods face difficulties in efficiently generating high-quality meshes when dealing with static or dynamic fluid-solid coupling computational domains with complex boundary shapes. To overcome the limitations of mesh-based numerical solution methods, meshless numerical solution methods have developed rapidly in engineering calculations. This method only uses point clouds to discretize the computational domain, and the topological constraints for generating point clouds are much smaller than those for discrete meshes, thus enabling flexible discretization of the computational domain.

Despite the progress made in the field, there remain notable gaps in the application of GFDM, especially the upwind GFDM, to the two-phase fluid-solid coupling of porous media. Current research on meshless methods in porous media tends to focus more on geomechanics and solid mechanics, with relatively limited attention paid to reservoir seepage problems, and even less to scenarios involving two-phase flow. While efforts have been made to integrate upwind schemes with GFDM for addressing convection-dominated flow issues, there is still a lack of systematic analysis regarding how this integrated approach performs in the context of two-phase fluid-solid coupling.

Another key gap lies in the application of GFDM to fractured porous media—a critical area for the development of low-permeability and tight reservoirs—where research remains insufficient. Additionally, there is significant room for improvement in the computational efficiency of GFDM when applied to large-scale engineering models, particularly through the optimization of node layout and the refinement of influence domain selection. Moreover, the potential of combining GFDM with emerging technologies like artificial intelligence to enhance its adaptability and accuracy in complex scenarios has not yet been fully explored, leaving a valuable avenue for future investigation.

Against this backdrop, this review aims to systematically synthesize the existing research progress on the application of upwind GFDM in the two-phase fluid-solid coupling of porous media. By providing a comprehensive overview of the current state of knowledge, identifying key challenges, and outlining promising directions for further study, this work seeks to lay a foundation for the development of general meshless numerical simulators for porous media flow, while also offering new insights to support the optimization of oil and gas exploitation strategies and environmental protection efforts.

2 Fluid-Solid Coupling Problems in Porous Media

Terzaghi [1] first considered the mechanism of fluid flow and medium deformation in deformable, saturated porous media, proposed the concept of effective stress, and established a one-dimensional consolidation model. Biot [2, 3] assumed that the material is isotropic and has linear elastic small deformation characteristics, the pore space of the material is saturated with incompressible fluid, and the fluid flow in the pore skeleton satisfies Darcy's law. On the basis of this assumption, a relatively complete three-dimensional consolidation theory was established. Subsequently, Biot extended this theory to the analysis of the coupling effect between fluid and pore pressure in anisotropic porous media [4] and dynamic analysis [5]. Since then, the development of porous media fluid-solid coupling theory has begun to consider the characteristics of different porous materials and establish corresponding fluid-solid coupling mathematical models based on these characteristics. Lubinski [6] and Geertsma [7] both discussed the Biot equation in the elastic theory of porous media. Zienkiewicz [8] proposed the generalized Biot theory considering geometric and material nonlinearity. Oda [9] used the fracture geometric tensor to express the relationship between rock mass seepage and deformation. Savage [10] applied Biot's three-dimensional consolidation theory to isotropic poroelastic media. Li et al. [11] considered the compressibility of solid and fluid phases, the capillary pressure between fluid phases, derived the fluid-solid coupling control equation on the assumption that fluid flow conforms to Darcy's law, and discussed the numerical solution method of the two-phase immiscible fluid-solid coupling model in detail. Li et al. [12-14] discussed the structure-mechanics interaction problem considering the consolidation effect of saturated soil and used the non-classical continuum finite element method for local strain analysis. Zhang et al. [15, 16] extended the generalized Biot formula established by Zienkiewicz et al. to the study of nonlinear problems of saturated soil consolidation.

With the development of the petroleum industry and the increasing demand for solving complex petroleum engineering problems, the research on fluid-solid coupling in porous media has attracted widespread attention in the fields of petroleum drilling, exploitation, and development. In the process of oil and gas exploitation, changes in pore fluid pressure will affect the physical properties of the rock matrix such as porosity and permeability, thereby affecting the migration of fluids in underground reservoirs. By simulating the interaction between pore fluid and rock skeleton, the productivity changes, seepage laws, pressure distribution, etc. of the reservoir can be predicted, providing a scientific basis for reservoir management and decision-making.

Lewis [17, 18] first considered disasters such as land subsidence caused by oil and gas exploitation, established an underground fluid-solid coupling model, and analyzed the influence of the coupling effect between pore fluid and rock mechanism on oil and gas production. Subsequently, the theoretical research on fluid-solid coupling in porous media incorporated the consideration of temperature field. Lewis, Wong, Settar et al. [19-22] have done a lot of innovative work on the thermo-hydro-mechanical coupling theory and its engineering applications in reservoir engineering. Tortike, Ali et al. [23-25] considered the development mode of steam thermal recovery in heavy oil reservoirs and established a thermo-hydro-mechanical coupling mathematical model considering oil, gas, and water phases. Neaupane et al. [26] established a thermo-hydro-mechanical coupling model in fractured rock masses, used the finite element method for solution and analysis, and used the model to discuss wellbore stability. Bear [27] studied the correlation between regional stress, temperature, and changes in rock permeability during geothermal exploitation. Faquhar [28] found that stress changes can cause changes in rock elastic modulus and permeability, and there is a linear relationship between Young's modulus and permeability. Bouteca [29] found through experiments that the volume coefficient of sandstone is a function of pore pressure and confining pressure. Fung [30] regarded geotechnical deformation as elastoplastic deformation, extended the two-dimensional isothermal seepage fluid-solid coupling model to a thermal recovery model, and solved it using the explicit alternating method. Sukirnan [31] considered the three phases of oil, gas, and water, and the elastoplastic characteristics of rocks, used the Mohr-Coulomb yield criterion to judge the failure situation, solved the fully coupled fluid-solid control equation using the finite element method and implicit iteration method, and used the results to simulate the surface subsidence phenomenon. Gutierrez [32] compared the results of conventional reservoir numerical simulation and fluid-solid coupled reservoir numerical simulation when predicting the temporal and spatial changes of fluid pressure, proving that the former cannot analyze the actual situation of reservoir compression only by using the rock compressibility coefficient.

In the 1980s and 1990s, domestic research institutions and scholars also began to apply the porous media fluid-solid coupling theory and model to solve practical problems in reservoir development and management. Ran et al. [33-35] established a mathematical model of multiphase fluid-solid coupling in underground reservoirs, which considers changes in parameters such as permeability and porosity, and uses the alternating iteration of finite difference and finite element for solution. The original model (uncoupled), elastic model, and plastic model were used for verification respectively, indicating that the coupling effect and the selection of constitutive model have a significant impact on the simulation results. Dong et al. [36-37] et al. considered factors such as elastoplastic deformation and creep, and established a mathematical model of fluid flow in variably saturated reservoirs. The model uses the finite element method to solve the unknowns of displacement and pore pressure, and the effectiveness of the model is verified through a single well production example. Xue et al. [38-39] established a mathematical model of seepage and geomechanics coupling in immiscible saturated porous media, derived and solved the finite element calculation format of the model using the decoupling method, and studied the coupling effect between pore fluid and stress field in the near-wellbore region. Liu [40] established a fluid-solid coupling mathematical model for low-permeability reservoirs considering the threshold pressure gradient and gave the numerical solution. Xu [41] studied the fluid-solid coupling mechanism in loaded formations under a two-dimensional stress field and obtained the analytical solutions of pore pressure and medium stress under coupled conditions. Wang [42] established a thermo-hydro-mechanical coupling mathematical model using basic laws such as mass (energy) conservation and thermodynamics, and derived the corresponding finite element calculation format. Xiong [43] took permeability, porosity, and compressibility as functions of pressure and established a fluid-solid coupling mathematical model for multiphase fluid flow in deformable porous media. Fan et al. [44] used the finite difference method to provide a coupled solution for reservoir seepage and elastic deformation of geotechnical interfaces, studied the time changes of strain, porosity, and permeability, and calculated the significant changes of parameters near the wellhead with time.

Fractures are the main seepage and production channels in low-permeability or tight reservoirs. In such unconventional reservoirs, the medium deformation effect will cause changes in fracture and matrix seepage parameters, which will have a great impact on production. Therefore, scholars in the industry have established various fluid-solid coupling mathematical models considering fracture dynamic behavior in real fractured reservoirs. Ji et al. [45] used the finite element method to solve the established fluid-solid coupling mathematical model of single-phase flow in dual media. Li et al., Xu et al. [46, 47] considered the nonlinear fluid-solid coupling seepage mechanism in which seepage parameters change with effective stress, established a fluid-solid coupling seepage model for deformable dual-porosity media, and gave the three-dimensional finite element calculation format. Liu [48, 49] established an equivalent continuum model suitable for seepage in fractured sandstone reservoirs, derived the equivalent processing method of seepage field and geotechnical deformation field parameters considering the influence of fractures, and gave the coupled calculation method based on the finite difference method and finite element method.

It should be noted that as an innovative research that first applies the meshless method to the field of reservoir numerical simulation, this paper only considers the two-phase fluid-solid coupling problem in general porous media, aiming to lay a sufficient theoretical foundation for the subsequent meshless method research considering fracture dynamic behavior in fractured porous media.

3 Mesh-Based Numerical Methods

So far, the main numerical solution methods for the fluid-solid coupling problem of oil-water two-phase flow in porous media include the Finite Difference Method (FDM) [50], Finite Element Method (FEM) [51], and Finite Volume Method (FVM) [52, 53].

A brief introduction to the three methods and their advantages and disadvantages is as follows:

(1) Finite Difference Method (FDM): The finite difference method approximates the differential equation at discrete grid points. Using methods such as Taylor series expansion, the derivatives of the differential equation are expressed by the difference quotients of the unknown function values at the grid nodes, thereby converting the differential equation into a system of algebraic equations, which can be solved numerically by iterative methods. The finite difference method is relatively easy to implement, but its accuracy is easily limited by grid density and step size, and it is necessary to select an appropriate grid size and step size to balance computational efficiency and accuracy; moreover, this method is not suitable for handling complex geometric shapes and boundary conditions, especially high-dimensional problems. In addition, when dealing with high-order differential equations and high-gradient problems, the finite difference method may require more complex techniques and larger computational load.

(2) Finite Element Method (FEM): The finite element method divides the computational domain into a finite number of interconnected but non-overlapping elements. Select appropriate nodes within each element as interpolation points to construct mathematical interpolation functions. The differential equation is expressed using the interpolation functions of variables. Then, the discrete format of the differential equation is obtained through the weighted residual method and variation principle, thereby obtaining the algebraic equations on each element. Finally, the overall system of algebraic equations is established by integrating the relationships between elements, and the numerical solution can be obtained by solving this linear system of algebraic equations. The finite element method has better numerical stability and accuracy when dealing with high-dimensional problems and high-order differential equations, so it is widely used in numerical simulation and analysis in fields such as structural mechanics, heat conduction, and fluid mechanics. However, when dealing with complex geometric shapes and nonlinear problems, the computational load of the finite element method is usually large.

(3) Finite Volume Method (FVM): The finite volume method divides the computational domain into a finite number of volume elements. A set of discrete equations is obtained by integrating the conservation equation within each volume

element, where the unknowns are the values of the dependent variables at the grid nodes. Then, the overall system of equations is established by calculating the fluxes between adjacent volume elements, and the numerical solution is finally obtained by solving this system of equations. The finite volume method naturally associates physical quantities with control volumes, making it easier to understand and analyze physical processes; it is suitable for handling partial differential equations in conservative form and can accurately maintain conserved quantities. Therefore, the finite volume method is often used in numerical simulation and analysis of problems such as fluid dynamics and heat and mass transfer. However, the finite volume method may face computational challenges when dealing with nonlinear problems and highly non-uniform grids, that is, it requires more memory and computing resources, and the computational load is large.

It can be seen that the above methods all need to first divide the computational domain into a series of meshes. For example, the finite element method needs to divide elements, the finite difference method uses grids, and the finite volume method divides into volume elements. The nodes on the meshes in the computational domain are interconnected through the meshes themselves, forming the application basis of the above traditional mesh-based numerical solution methods. However, the mesh system may face some challenges and difficulties in the solution process of the porous media fluid-solid coupling numerical model [54-56]:

(1) Difficult mesh generation: Mesh-based methods require mesh generation, and due to the complex geometric shape of the porous media computational domain, generating suitable meshes is often very difficult and time-consuming.

(2) Mesh subdivision incompatibility: The flow and phase change properties in porous media may lead to mesh subdivision incompatibility. If the mesh is distorted during the flow process, the mesh quality will decrease, which will seriously affect the accuracy of the solution, reduce computational efficiency, and even lead to computational failure. In this case, mesh reconstruction or adaptive mesh adjustment is required, which will have a significant impact on computational accuracy and speed.

(3) Numerical dissipation: Mesh-based methods will introduce numerical dissipation during the calculation process, leading to a decrease in the accuracy of the numerical solution, and it is difficult to capture small-scale phenomena.

4 Meshless Numerical Solution Methods

Meshless methods adopt a node-based discretization method, avoiding the difficulties of mesh generation and subdivision faced by the above traditional mesh-based numerical methods:

(1) Meshless methods only use point clouds to discretize the computational domain, and the topological constraints for generating point clouds are much smaller than those for mesh discretization. The point clouds in meshless methods can use simple data structures (such as arrays or linked lists) to store node position information. In contrast, traditional mesh-based numerical methods need to use two-dimensional or three-dimensional arrays to store the properties and state variables of mesh elements, which increases the complexity of the data structure.

(2) Meshless methods can adaptively move and adjust nodes in the porous media computational domain to adapt to the properties of flow and phase change, thereby avoiding the problem

of mesh subdivision incompatibility.

(3) Since meshless methods do not rely on meshes, there is no numerical dissipation effect caused by mesh size and shape, and the details of flow and phase change can be captured more accurately.

The research on meshless methods originated in the 1970s. Compared with the finite element method, which was the mainstream numerical solution method at that time, meshless methods had not attracted widespread attention from scholars, and the research progress was slow. Lucy [57] first proposed the Smoothed Particle Hydrodynamics (SPH) method and successfully introduced it to boundary-free astrophysics problems. This method uses discrete particles to describe fluids that are continuously distributed macroscopically but still particles microscopically, and is regarded as the first meshless method in the industry. However, its computational accuracy and stability are not satisfactory. Subsequently, Libersky et al. [58] extended the SPH method to the study of diaphragm impact problems, and Chen et al. [59] improved the SPH method by introducing a viscosity coefficient, which improved its stability. Since then, the research on meshless methods has almost stagnated. In the 1990s, Nayroles et al. [60] applied the Moving Least Square (MLS) method to the Galerkin method to form the Diffuse Element Method (DFM). This method only needs a series of discrete points and boundaries to describe the solution domain, initially possessing the characteristics of meshless methods. Subsequently, Belyschko et al. [61] considered some terms in the shape function derivative expression that were ignored by Nayroles based on MLS, and used Lagrange multipliers to handle intrinsic boundary conditions, forming the Element-free Galerkin Method (EFGM). Compared with the SPH method, this method has a slightly higher computational cost but higher computational accuracy and stability. EFGM was later applied to dynamic crack propagation simulation [62, 63] and three-dimensional impact analysis [64], and achieved good simulation results. Since then, meshless methods have been widely used in various scientific and engineering problems [65-69].

At present, the research directions of meshless methods are divided into boundary type and domain type according to whether the interpolation basis function can satisfy the control equation. Among them, the boundary-type meshless method is similar to the boundary element method. On the basis of inheriting the advantage of the boundary element method in reducing the dimension of the problem to be solved, it only needs boundary collocation points to realize simulation calculation, and avoids the complex singular and nearly singular integral calculations in the boundary element method. At present, popular boundary-type meshless methods include the Boundary Knot Method (BKM) [70], Method of Fundamental Solutions (MFS) [71], Singular Boundary Method (SBM) [72, 73], Boundary Element-free Method (BEFM) [74], Regularized Meshless Method (RMM) [75], etc. The above methods have achieved varying degrees of success, but problems such as the easy generation of full-rank coefficient matrices and the dependence on the selection of interpolation basis functions (fundamental solutions) limit the application and development of such methods in practical engineering problems [76]. Different from boundary-type methods, localized domain methods introduce the concept of computational domain localization, that is, the entire computational domain can be divided into multiple subdomains, so that the linear system generated by discretizing the control equations will be a sparse matrix. Therefore, a variety of different effective solution methods can be used to obtain numerical solutions accurately and quickly. Moreover, such methods do not depend on the selection of interpolation basis functions (fundamental solutions), so they can handle initial and boundary value problems of various partial differential equations and have broader application prospects. At present, commonly used localized domain methods mainly include the Element Free Galerkin Method (EFGM) [77], Diffuse Element Method (DEM) [78], Smoothed Particle Hydrodynamics (SPH) [79], and Generalized Finite Difference Method (GFDM) [80], etc.

In the field of geomechanics research, Zhou et al., Kou et al. [81-85] were the first to introduce EFGM into the research of fracture mechanics problems in geotechnical engineering, and predicted arch dam cracking and groundwater flow through EFGM simulation. Ge et al. [86] introduced EFGM into seepage calculation with free surfaces. Zhang and Pang [87, 88] used EFGM to calculate the bending problem of foundation slabs. Wang et al. [89, 90] regarded soil as an ideal medium and used EFGM to solve the consolidation equation. Zhang [91] first gave the error analysis of the discrete equation of consolidation EFGM, indicating that the refinement degree of the mesh integration structure has a great influence on computational accuracy and stability. Pang [92-94] used EFGM to simulate slope excavation in geotechnical engineering, contact (friction) between two objects, and discontinuous surfaces.

In the field of solid mechanics research, Yang [95] constructed an incremental FEM-EFGM coupled solution technology using surface force coupling technology, which can solve general solid mechanics problems. He et al. [96-98] applied the FEM-EFGM coupled solution technology to the analysis of dynamic crack propagation problems. Liu et al. [99] established the calculation formula of the FEM-EFGM method, and verified the feasibility of introducing essential boundary conditions through Lagrange multipliers through examples. Wang et al. [100-102] used EFGM to simulate crack propagation and plate bending. Zhang et al. [103-106] established frameworks based on the compactly supported weighted residual meshless method and the least square collocation meshless method respectively, and the latter has considerable computational accuracy, efficiency, and stability. Cai et al. [107-109] proposed a meshless natural element method for solving plane problems of elasticity based on the natural neighbor approximation displacement function and applied it to underground engineering problems. Lu et al. [110, 111] realized the solution error estimation of the natural element method based on the Z-Z method. Dai [112] derived an algorithm for three-dimensional natural neighbor coordinates and their derivatives based on the Lasserre convex polyhedron volume formula, and gave the flow chart of the three-dimensional natural element method algorithm. Luan, Tian et al. [113-116] combined the element-free method with the manifold method based on finite cover to form the finite cover element-free method, which was used to analyze the fracture characteristics and crack propagation of complex rock masses.

In the field of fluid mechanics research, Qiu et al. [117] used EFGM to numerically simulate the two-dimensional incompressible viscous flow around a cylinder. Fang et al. [118] used the SPH method to simulate liquid sloshing with a free liquid surface in a liquid storage container. Wang et al. [119] developed a set of meshless algorithms for solving unsteady flows. This method automatically generates point clouds by filling and placing points according to regions, and the comparison between the calculation results and experimental measurement results verifies the effectiveness and practicality of the method.

For underground seepage problems, Zeng et al. [120, 121] discussed single-phase seepage problems using EFGM. Huang and Shu [122, 123] applied EFGM and the Moving Least Squares (MLS) meshless method to the solution of reservoir seepage problems and derived the meshless discrete format for oil-water two-phase seepage problems. Li [124] first introduced the meshless method into numerical well testing, and used the Minimum Weighted Least Squares (MWLS) method combined with EFGM to solve the well test interpretation model considering complex boundaries, heterogeneity, and oil-water two-phase fault-block reservoirs, achieving good results in computational accuracy and efficiency. Rao et al. [125] used the MLS method combined with the MWLS method to study the water-gas flow problem in dual-media shale gas reservoir models. The reservoir example with complex boundary shapes verified the high accuracy, stability, and convergence of the proposed method, revealing the great application potential of the meshless method in reservoir seepage problems.

Combined with the above research status of meshless methods, it can be seen that as a new type of numerical calculation method, the research of meshless methods is mainly concentrated in the fields of geomechanics and solid mechanics. As a typical porous medium, the application research of meshless methods on the seepage mechanism of underground reservoirs is slow and the results are few. The complexity, nonlinearity, and multi-scale characteristics of underground reservoir seepage problems limit the application of traditional numerical methods. However, the mesh-independent characteristics of meshless methods will provide new possibilities for more flexible, accurate, and convenient seepage prediction. Therefore, promoting the application research of meshless methods in porous media seepage problems may provide important references for in-depth understanding of porous media seepage mechanisms, optimizing energy development strategies, and improving energy exploitation efficiency.

5 the Generalized Finite Difference Method

GFDM is one of the most promising methods among meshless methods [126, 127]. Developed from the classical finite difference method, GFDM expresses the partial derivatives of various orders of unknown quantities in the control equation as a linear combination of the function values of adjacent nodes in the subdomain based on the multivariate function Taylor series expansion and weighted least square fitting in the subdomain, overcoming the dependence of traditional FDM on meshes. GFDM can handle problems with discontinuities, singular points, and high-order derivatives. In addition, the consistency in principle between GFDM and FDM provides convenience for them to use similar nonlinear solution strategies (such as the Newton iteration method). That is, in practical applications, there is no need to write a dedicated NR solver for GFDM, which obviously reduces the technical difficulty of forming a general meshless numerical simulator for porous media multiphase flow based on GFDM.

At present, GFDM is developing rapidly and is widely used in solving various scientific and engineering problems, including shallow water equations [128], high-order partial differential equations [129], transient heat conduction analysis [130], stress analysis [131], water wave interaction [132], inverse heat source problems [133], seismic wave propagation problems [134], coupled thermo-elasticity problems [135-137], stochastic analysis of groundwater flow [138], and some typical differential equation problems (such as unsteady Burgers' equations [139, 140], nonlinear convection-diffusion equations [141], time-fractional diffusion equations [142]). The generalized finite difference method only needs to arrange a set of nodes in the computational domain to achieve accurate solution of the control equation, saving the time-consuming and laborious mesh division and numerical integration of the complex geometric features of the computational domain in the finite element method, finite difference method, and boundary element method. It is an efficient and high-precision numerical modeling method.

For flow problems, when dealing with certain physical parameters in the control equation, it is often necessary to adopt an upwind scheme. In meshless methods, the upwind scheme is generally realized by modifying the influence domain of nodes, including the upwind influence domain method that moves the central node to the upstream direction [143] and the partial influence domain method that includes more upstream nodes in the influence domain of the central node [144]. However, due to the possible complexity of the actual flow field, it is difficult to form a stable upwind effect by modifying the node influence domain, and the computational accuracy is also difficult to be well guaranteed. Sridar and Balakrishnan [145] proposed a finite difference method for computational fluid dynamics based on the upwind least square method. Saucedo-Zendejo et al. [146] used the upwind GFDM to solve the three-dimensional free surface flow in the mold filling process. Michel et al. [147] applied the upwind GFDM to simulate the solution mining process at the micro and macro scales. Shao et al. [148] used the upwind GFDM to solve the Stokes interface problem.

Immiscible two-phase flow is a basic case of porous media flow problems, such as oil-water two-phase flow in underground reservoirs [149-151]. The relative phase permeability in the porous media two-phase flow equation is a typical physical parameter that needs to adopt the upwind scheme. Rao et al. [152] applied the upwind GFDM to the modeling of single-phase heat and mass transfer in porous media, adopted a sequential coupled format, and proposed a high-precision convection-diffusion equation solution method based on the upwind GFDM, which also means the great application potential of the upwind GFDM in porous media seepage problems. Subsequently, Rao et al. [153, 154] applied the generalized finite difference method (GFDM) based on the fully implicit scheme to oil-water two-phase flow. To enable GFDM to handle singular source-sink terms, Rao et al. [155] further developed an integral-form extended finite volume method (EFVM) based on GFDM. This method can maintain local mass conservation in the sense of least squares and directly invoke the nonlinear solvers for various seepage models embedded in existing simulators. Currently, the extended finite volume method has been successfully applied to black-oil models and compositional models [156, 157]. In addition, Rao et al. [158] extended EFVM to fractured hydrocarbon reservoirs and developed a meshless discrete fracture model.

6 Conclusion

This review systematically examines the application of upwind Generalized Finite Difference Method (GFDM) in the two-phase fluid-solid coupling of porous media, synthesizing the evolution of relevant theories, numerical methods, and current research progress, while identifying key research gaps and future directions. The main conclusions are as follows:

First, the development of porous media fluid-solid coupling theory has progressed from basic consolidation models to complex multi-physics coupling frameworks. Starting with Terzaghi's one-dimensional consolidation model and Biot's three-dimensional consolidation theory, subsequent studies have integrated nonlinearity, anisotropy, thermal effects, and elastoplastic deformation, providing a solid theoretical basis for understanding the interaction between fluid flow and medium deformation in porous media. However, the practical application of these theories still relies heavily on efficient numerical methods, especially in addressing the complexity of underground reservoir scenarios.

Second, traditional mesh-based numerical methods (FDM, FEM, FVM) have played important roles in solving porous media fluid-solid coupling problems but are constrained by inherent limitations. Difficulties in mesh generation for complex domains, poor adaptability to dynamic flow and phase change processes, and the presence of numerical dissipation limit their performance in complex engineering applications. Meshless methods, by contrast, have emerged as a powerful alternative, with GFDM standing out due to its meshless nature, consistency with traditional FDM (facilitating the use of mature nonlinear solvers), and ability to handle discontinuities and high-order derivatives. The integration of upwind schemes with GFDM further enhances its stability and accuracy in solving convection-dominated flow problems, addressing a key shortcoming of traditional meshless methods in achieving reliable upwind effects.

Third, despite the promising potential of upwind GFDM, several critical research gaps persist. The application of GFDM in porous media has been primarily limited to single-phase seepage and general geomechanics, with insufficient focus on two-phase fluid-solid coupling—especially in fractured porous media, which are crucial for low-permeability oil and gas reservoirs. Additionally, the computational efficiency of GFDM for large-scale engineering models needs improvement, with opportunities for optimization in node layout and influence domain selection. Furthermore, the integration of GFDM with emerging technologies such as artificial intelligence to enable intelligent parameter optimization and adaptive simulation remains largely unexplored.

Looking ahead, future research should prioritize several directions. First, expanding upwind GFDM to fractured porous media, considering the dynamic behavior of fractures, to meet the practical needs of unconventional reservoir exploitation. Second, optimizing GFDM's computational parameters, including node distribution strategies and influence domain selection criteria, to reduce computational costs while maintaining solution accuracy. Third, exploring the fusion of GFDM with artificial intelligence technologies, such as machine learning for node layout optimization and deep learning for inverse problem solving, to develop more intelligent and efficient numerical simulation tools. Finally, conducting more engineering verification studies, comparing GFDM results with field test data, to enhance its reliability and promote its industrial application.

In summary, upwind GFDM provides a innovative and effective technical approach for addressing the complex two-phase fluid-solid coupling problems in porous media. This review not only consolidates the current state of research but also provides a roadmap for future developments, which will contribute to the advancement of porous media flow simulation technology, the optimization of oil and gas exploitation efficiency, and the protection of the ecological environment.

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