Advanced Energy Systems and Intelligent Geoscience (AESIG)
https://www.mospbs.com/journal/aesig

MOSP

• *Review* •

A Review of Numerical Simulation and Connectivity Analysis in Complex Media Reservoirs

Wentao Zhan[1,2,*], Yixin Zhang[3], Liang Pu[4], Zhaolin Pu[3]

[1] Western Research Institute, Yangtze University, Karamay 834000, China.

[2] Key Laboratory of Exploration Technologies for Oil and Gas Resources, Ministry of Education, Yangtze University, Wuhan 430100, China.

[3] College of Petroleum Engineering, Yangtze University, Wuhan 430100, China.

[4] Engineering Technology Supervision Center, Changqing Oilfield Company, Petro China, xian, shanxi, 710000, China.

[*]**Corresponding Author:** Wentao Zhan **Email:** zhanwt1996@163.com

Received: 29 December 2025 **Accepted:** 06 January 2026

Abstract: This paper reviews the key technologies and challenges in numerical reservoir simulation and inter-well connectivity analysis for complex media. Addressing the issues of complex fluid flow patterns and strong heterogeneity in deep-layer, deep-water, and unconventional reservoirs, this study systematically summarizes the advantages and disadvantages of three mainstream approaches: reservoir engineering, numerical simulation, and machine learning. Four core components of numerical reservoir simulation are analyzed, with a detailed comparison of various computational domain discretization techniques, including Cartesian, corner-point, PEBI grids, and meshless methods. Furthermore, the principles and applicability of six major numerical models (e.g., equivalent continuum, dual-porosity, and discrete fracture models) used to characterize complex media such as fractures and vugs are discussed in depth. Regarding inter-well connectivity analysis, the progress of traditional reservoir engineering methods—such as logging, well testing, and chemical analysis—as well as numerical methods like streamline models and physical connectivity network models are summarized. Finally, the paper identifies current technical deficiencies in handling anisotropy, characterization efficiency of complex geometries, and quantitative characterization of dynamic connectivity, providing directions for future research.

Keywords: Complex media reservoirs; Computational domain discretization; Numerical simulation; Inter-well connectivity

1 Introduction

Currently, the intensive exploitation of conventional oil and gas and the economic development of unconventional resources are effective means to address energy dilemmas. The exploration and development of unconventional oil and gas have achieved industrial-scale

DOI: https://doi.org/10.58244/aesig.253469

progress; their increasing production share has made them strategic resources for “stabilizing oil and increasing gas” within the global energy consumption landscape. As reservoirs are deeply buried underground, rapid and accurate dynamic simulation and prediction remain eternal themes for efficient oilfield development. However, as reservoir types become more complex—such as deep-layer, deep-water, and unconventional reservoirs with developed natural fractures and fracture-vug media—fluid flow patterns become highly variable, exhibiting strong heterogeneity and geological uncertainty.

At present, three main categories of methods exist: reservoir engineering, numerical simulation, and machine learning, each with specific strengths and weaknesses. Reservoir engineering methods are typically based on well-group or macroscopic reservoir models. Their advantage lies in direct integration with field applications, providing practical operational suggestions. While concise, their accuracy is limited by coarse assumptions and simplifications, making it difficult to accurately reflect reservoir complexity. Numerical simulation methods rely on fine-scale geological models and offer high precision through detailed physical modeling. However, they suffer from high computational costs and poor convergence, which can lead to unstable results in complex scenarios. Machine learning methods, as emerging proxy models, offer high speed and the ability to process large-scale data. Despite their rapid fitting capabilities, they require high-quality training data and lack interpretability due to their “black-box” nature. Consequently, technical methods that meet the practical field application requirements for complex media reservoirs still require refinement.

2 Discretization Methods in Numerical Reservoir Simulation

The discretization methods in numerical reservoir simulation are primarily categorized into grid-based and meshless approaches. As illustrated in Figure 1-1, grid-based discretization techniques simulate fluid flow by partitioning the reservoir into structured or unstructured grids. Structured grids are arranged in regular geometric patterns, such as orthogonal grids [4] and Cartesian grids [5]. Their advantages include high computational efficiency and numerical stability; however, they may exhibit insufficient flexibility when characterizing complex geological morphologies. For instance, the Pillar Grid is a widely utilized structured grid capable of modeling geological features such as faults, boundaries, and pinch-outs, though its non-orthogonality may compromise the accuracy of numerical simulations [6].

Conversely, unstructured grid technologies have gained prominence due to their inherent flexibility and adaptability to complex geological structures. These grids do not require grid nodes to follow a strict geometric sequence and include PEBI (Perpendicular Bisection) grids [7, 8], triangulated grids [9], and polygonal grids [10]. Unstructured grids offer superior flexibility in handling complex reservoir geometries, enabling a more precise approximation of reservoir boundaries and internal architectures.

Furthermore, meshless discretization utilizes nodes to represent the interior and boundaries of the computational domain. Unlike grids, nodes are points without predefined geometric structures, thereby liberating the simulation from the constraints of geometric topological structures. Meshless methods possess inherent advantages for discretizing computational domains with complex geometries, offering greater degrees of freedom

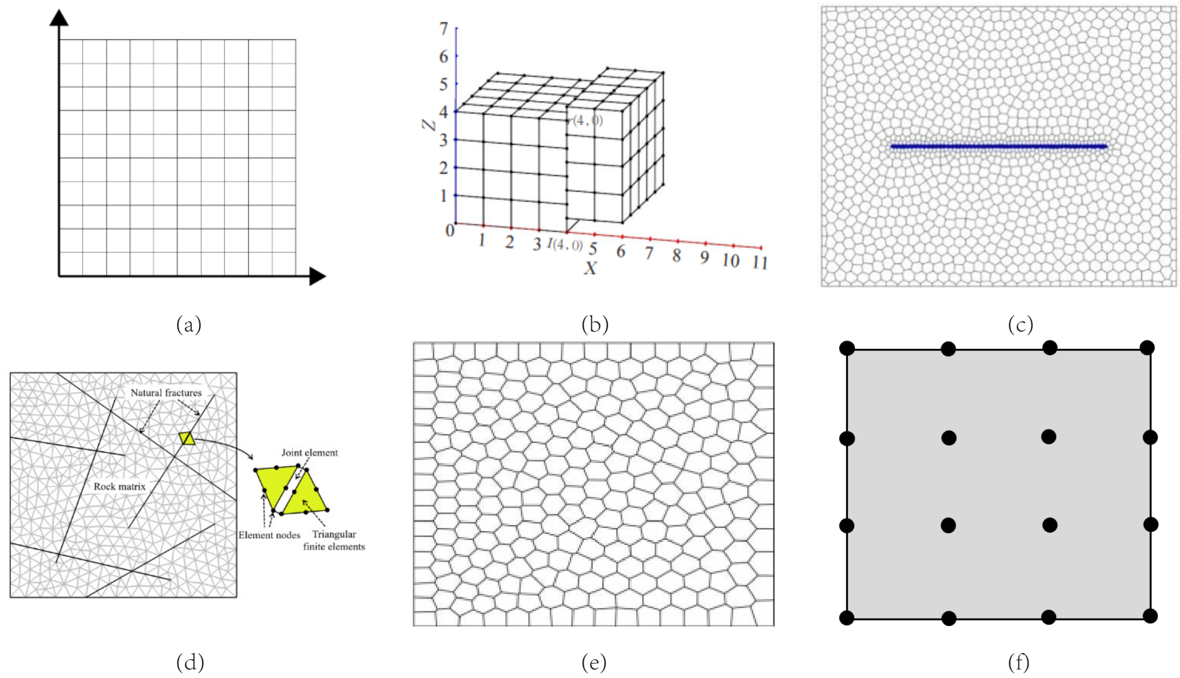

Fig.1 Discretization of reservoir computing domain

and flexibility compared to traditional grid-based discretization methods.

The Cartesian grid technology was among the earliest meshing methods adopted, providing a concise framework for numerical simulation by partitioning the reservoir into regular rectangular parallelepiped units. The advantages of this method lie in its rapid grid generation and high computational efficiency, making it suitable for simulating regular reservoirs and simple flow problems. However, when confronted with complex reservoir boundaries and geological features—such as fractures, pinch-outs, and faults—the Cartesian grid lacks sufficient flexibility for precise modeling. Furthermore, it exhibits certain limitations in handling non-vertical wells (e.g., horizontal and deviated wells) due to its inability to accurately characterize actual wellbore trajectories [13-15].

To overcome these limitations, corner-point grid technology was developed. Based on the eight vertices of a hexahedral unit, corner-point grids [16-20] form a “deformed” mesh structure through specific stretching and twisting operations to better accommodate the complex requirements of reservoir simulation. This technology’s strength lies in its high adaptability and flexibility, enabling a more precise reflection of complex boundaries and variable geological structures. Nevertheless, its non-orthogonality may compromise computational accuracy to some extent, particularly when well locations are not centered within the grid blocks.

Radial grid technology [21] was proposed to enhance simulation precision and adaptability. Built upon a cylindrical coordinate system, its structured partitioning aligns closely with the radial flow characteristics around the wellbore. Radial grids are particularly suited for simulating problems concentrated near injectors and producers, as they can accurately capture the actual fluid flow state. Moreover, the relatively minor constraints on grid volume allow for extremely fine discretization in the near-wellbore region to ensure precision, while grid volumes can be appropriately increased in areas distant from the well.

Streamline-based grid technology [21] evolved from streamline representation methods. By constructing and discretizing the fluid flow process along streamlines, it transforms complex 3D flow problems into a series of more manageable 1D problems. A significant advantage of this technique is its ability to simulate complex flow with simplified computational logic, thereby enhancing speed and accuracy. However, a limitation of streamline-based grids is their inability to reflect the real-time dynamic changes of reservoir fluids; current research primarily focuses on solving incompressible flow problems.

CVFE (Control Volume Finite Element) grid technology [21] utilizes a unique triangular partitioning approach to refine the discretization process. Constructed based on the vertices of triangular elements and the lines connecting edge midpoints to the geometric center (centroid), CVFE grids ensure favorable symmetry and continuity across the computational domain while preserving original formation features for subsequent reservoir analysis.

PEBI (Perpendicular Bisection) grid technology [22] represents a major advancement in the third generation of meshing techniques. It optimizes traditional 3D model construction through an innovative partitioning strategy based on face-edge connections of triangles. Specifically, it uses the circumcenters of triangles to construct polygonal units, ensuring that the interface of any two adjacent cells is perpendicular to the line connecting their centers. This perfectly satisfies the orthogonality requirements of the finite difference method, thereby improving simulation precision. Through refined node distribution, PEBI grids ensure alignment with key geological features such as faults, fractures, and horizontal wells, significantly reducing errors caused by geological discontinuities.

Meshless methods [23] characterize the entire domain using a set of freely distributed field nodes on the problem domain and its boundaries. These nodes do not form traditional mesh structures, thereby bypassing the constraints of topological relationships. Instead of pre-defining connectivity, they employ point-based approximation methods to construct unknown variable functions. This approach simplifies the computational workflow and enhances flexibility by eliminating initial meshing and subsequent remeshing operations. Furthermore, it mitigates the impacts of remeshing, such as enriching mass transport paths based on the influence domain or local point clouds, which helps eliminate the grid orientation effect (GOE). Zhao [24, 25] further advanced this by discretizing the reservoir into an equivalent physical connectivity network to intuitively describe connectivity. This method, inheriting meshless characteristics, describes multi-scale complex geometries by defining flow parameters on the connection system, thereby flexibly characterizing flow behavior and revealing inter-node interactions.

In summary, while Cartesian grids are widely used for their simplicity and efficiency, they are limited by their adaptability to complex structures. Corner-point grids offer higher flexibility at a higher computational cost. Radial grids focus on near-wellbore radial flow, while streamline grids simplify 3D problems for speed. CVFE and PEBI grids optimize discretization and orthogonality for complex conditions. Finally, meshless methods and connectivity networks provide the greatest degrees of freedom for handling complex geometries and dynamic transitions, offering a more intuitive foundation for inter-well connectivity analysis.

3 Numerical Simulation Methods for Reservoirs with Complex Media

The reservoirs with complex media investigated in this study primarily refer to those characterized by fractures (both hydraulic and natural), vugs, complex boundaries, and faults. To investigate the seepage characteristics within such complex media, six categories of numerical models are primarily employed: the Equivalent Continuum Model, the Dual-Medium Model, the Discrete Fracture Model (DFM), the (Projected) Embedded Discrete Fracture Model (EDFM), the Discrete Fracture-Vug Network Model, and Meshless Numerical Simulation. These models are systematically introduced in the following sections.

3.1 Equivalent Continuum Model

The equivalent permeability model was first proposed by Snow [26] in 1968, which treats fractures and the rock matrix as a unified continuum. This model assumes that the fluid exchange between the matrix and fractures reaches a state of local equilibrium. By equivalently averaging the fracture permeability across the entire reservoir, the system is simulated as an anisotropic continuous medium characterized by a permeability tensor. The primary advantages of this approach include model simplicity and low computational complexity, making it suitable for large-scale reservoir simulations, particularly in homogeneous reservoirs with low fracture density.

Oda [27] assumed that the average pressure gradient within the fractures remains consistent with that in the adjacent matrix. By simplifying fractures as line sources, the equivalent permeability of the matrix was derived using the Boundary Element Method (BEM). In contrast, Lough et al. [28] determined the corresponding equivalent permeability based on the assumption of steady-state crossflow between the matrix and fractures. Lee et al. [29] proposed a hierarchical model, the core of which involves establishing a matrix grid for the reservoir and classifying fractures into small, medium, and large scales based on the number of grid blocks they intersect; they advocated for applying the equivalent continuum model specifically to small-scale fractures. Additionally, Fang [30] further derived the formulae for the equivalent permeability tensor by treating fractures as an equivalent continuum.

In practice, the equivalent continuum approach discretizes fractured reservoirs into anisotropic reservoirs. Common methods for handling such anisotropy include spatial coordinate transformation [31, 32], Two-Point Flux Approximation (TPFA) [33, 34], and Multi-Point Flux Approximation (MPFA) [35-38].

Spatial transformation theory converts anisotropic problems into isotropic ones via coordinate mapping; while simple and efficient, it increases the complexity of boundary conditions and lacks general applicability, as shown in Figure 1-2. The TPFA method, within the finite difference framework, estimates the flux across interfaces using the pressure values of two adjacent grid cells to determine transmissibility. However, this method is only applicable to orthogonal grids and diagonal permeability tensors, failing to handle unstructured grids or full tensors. As illustrated in Figure 1-3, the MPFA method is based on the control volume method under the conditions of flux and pressure continuity. It partitions adjacent grids into several interaction regions to approximate interface flux through multiple surrounding regions, thereby determining the transmissibility. Although MPFA is applicable to unstructured grids and full permeability tensors, it is characterized by higher computational complexity and relatively lower numerical stability.

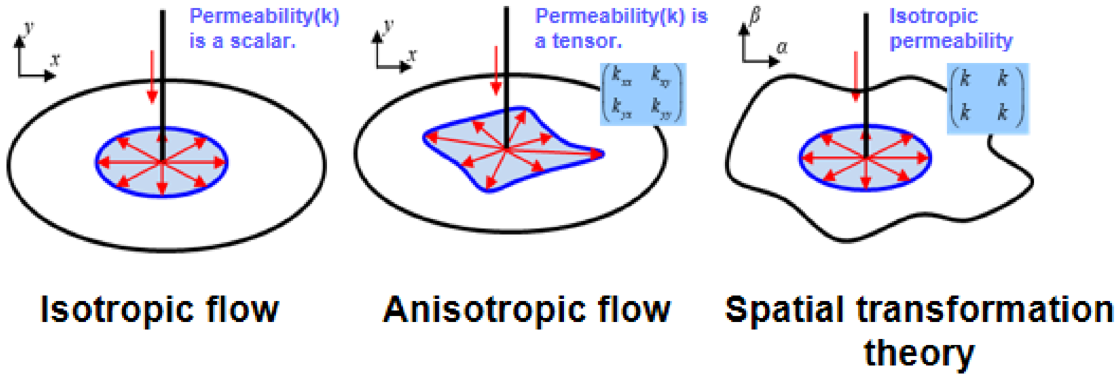

Fig.2 Space transformation theory

Despite its computational simplicity, the equivalent permeability model fails to reflect the actual fluid flow variations within the reservoir and struggles to accurately simulate the detailed characteristics of local flow fields. Its computational precision significantly diminishes particularly in scenarios with highly non-uniform fracture distributions. Furthermore, since the model is predicated on the concept of equivalent averaging, it may overlook the granular impacts of fractures on the overall seepage field, thereby constraining its applicability in the simulation of reservoirs with complex fracture networks.

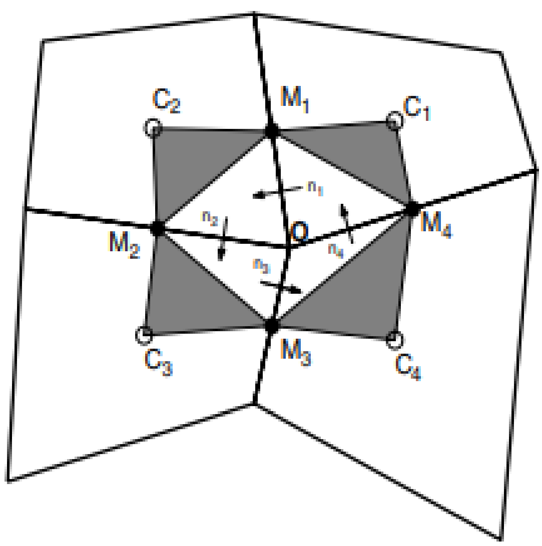

Fig.3 Multipoint flux approximation [39]

3.2 Dual-Medium Model

The dual-medium model is a traditional approach for characterizing fractured reservoirs, first proposed by Barenblatt et al. [40] in 1960. In this model, the flow intensity between the matrix and fractures is assumed to be directly proportional to the pressure differential and inversely proportional to the fluid viscosity. Subsequently, Warren and Root [41] and Kazemi [42] developed various dual-medium models based on different fracture distribution characteristics and applied them to the petroleum industry.

These models conceptualize the reservoir as a composite of matrix and fractures. The matrix serves as the primary storage space for fluids, with a porosity significantly higher than that of the fractures; conversely, the fractures act as the principal conduits for fluid migration, exhibiting a permeability much greater than that of the matrix. Consequently, in fractured oil and gas reservoirs, the matrix and fractures form two distinct yet interconnected flow systems. In the process of numerical simulation, two sets of porosity and permeability can be defined at every point in space; thus, each spatial point is assigned two pressures and two velocities. Dual-medium models can be further categorized into dual-permeability models, dual-porosity models, and dual-porosity dual-permeability (DPDP) models. These models provide an effective methodology for understanding fluid dynamics within complex fractured reservoirs.

Fracture system:

$$\nabla \cdot \left[\frac{k_m k_{r\sigma}}{B_\sigma \mu_\sigma} \nabla p_{\sigma,m} \right] + q_{osi}\delta = \frac{\partial}{\partial t}\left(\frac{\phi s_{\sigma,m}}{B_\sigma} \right) + \tau_{mf},\ \sigma = o, w$$

Matrix system:

$$\nabla \cdot \left[\frac{k_f k_{r\sigma}}{B_\sigma \mu_\sigma} \nabla p_{\sigma,f} \right] = \frac{\partial}{\partial t}\left(\frac{\phi s_{\sigma,f}}{B_\sigma} \right) - \tau_{mf},\ \sigma = o, w$$

Matrix-to-fracture crossflow:

$$\tau_{mf} = \sigma \frac{k_m}{\mu}(p_m - p_f$$

Therein, σ denotes the shape factor, which was initially defined in the dual-medium model by Warren and Root [41] to characterize the geometric contact relationship between the matrix and fractures. This model employs the analytical solution of spherical blocks to analyze late-time crossflow and provides an in-depth investigation of the dual-medium framework. It assumes that continuously and uniformly distributed fractures partition the matrix blocks into two isotropic continua: the fracture system and the matrix system (as illustrated in Figure 1-4). The interrelationship between these systems is described by assuming fluid exchange between the fractures and the matrix. Specifically, the fracture system exhibits high permeability but low storage capacity; conversely, the matrix system possesses high storage capacity but low permeability. This characteristic enables mutual interference between the two systems during the flow process, thereby influencing the overall hydrocarbon migration and recovery efficiency. Subsequently, Coats [42] and Aziz [43] refined the shape factor within this model,

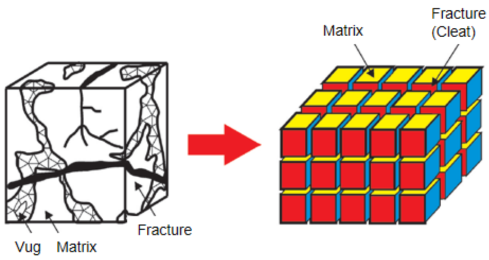

Fig.4 Warren-Root dual porosity model [41]

further enhancing its applicability and accuracy, and facilitating its extensive implementation in the simulation of fractured reservoirs.

Although the dual-medium model offers certain advantages in describing fractured reservoirs, its premise—assuming that fractures uniformly partition the matrix—constrains the model's capability to effectively characterize actual fracture geometric features. Specifically, the model fails to accurately reflect the height, length, width, and spatial distribution of fractures. This simplification often results in a significant discrepancy between the simulated flow behavior and actual field conditions in fractured reservoirs.

Kazemi et al. [44] refined the Barenblatt and Warren-Root models by proposing the layered dual-medium model. This model conceptualizes the dual-medium system as a set of layered matrices partitioned by parallel bedding planes and fractures, characterized by alternating horizontal fractures and horizontal bedrock (as illustrated in Figure 1-5). Through this simplification, the model better captures the flow characteristics of layered reservoirs. Subsequently, James et al. [45] introduced an exchange function dependent on the pressure gradient, further optimizing the flow description within the layered dual-medium model. Additionally, Deswaan [46] developed the Deswaan spherical dual-medium model; building upon the Warren-Root framework, this model simplifies matrix blocks into regularly arranged spheres,

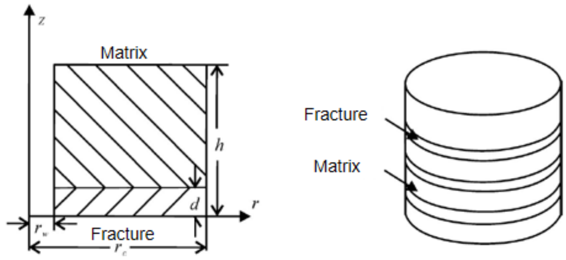

Fig.5 Schematic diagram of Kazemi laminar dual porosity model

with fractures represented by the interstitial spaces between these spheres. By converting the matrix morphology into a spherical geometry, this model provides a more intuitive representation of the fluid storage characteristics of the matrix.

Multi-medium models represent a further refinement and development of dual-medium models, reclassifying the reservoir based on the heterogeneity of fractures and the matrix to form various types of multi-medium systems. Liu et al. [47] proposed a triple-medium model to accommodate more complex flow scenarios, which was subsequently refined by Abdassah et al. [48] to enhance its flexibility and accuracy. Furthermore, Wu et al. [49] developed a triple-medium model consisting of the matrix, large-scale fractures, and small-scale fractures, conceptualizing the fluid flow space as an idealized superposition of three continua with distinct characteristic parameters. Each medium system remains independent yet interacts with others, thereby providing a more comprehensive description of the flow processes in fractured reservoirs. During the evolution of multi-medium research, Pruess et al. [50], inspired by the heat transfer processes in fractured reservoirs, introduced the Multiple Interacting Continua (MINC) model (as illustrated in Figure 1-6). Based on the dual-medium concept, the MINC model further subdivides the matrix into multiple nested elements. Each nested element shares a uniform pressure value, and the pressure distribution across these elements is calculated based on one-dimensional linear analytical solutions. The primary advantage of this model lies in its ability to elaborately describe the matrix pressure distribution in low-permeability reservoirs while significantly reducing computational overhead and enhancing calculation speed without compromising model precision.

The MINC model proposed by Pruess and Narasimhan effectively simulates the unsteady-state crossflow characteristics between the matrix and fractures by partitioning matrix grid blocks into multiple nested sub-grids. The model stipulates that fluid flows sequentially from the innermost sub-grid to the outermost layer, and ultimately into the fracture grid, thereby precisely capturing the flow behavior within the matrix. Building upon this, Gilman [51] developed a reservoir simulator based on the MINC model, which further partitioned each matrix block into nested rectangular grids and incorporated gravity effects. This advancement rendered the simulation results more consistent with field realities, particularly in reservoirs characterized by gravity segregation. Research by Wu and Pruess [52] demonstrated that the MINC model achieves higher computational precision than traditional dual-medium models, the latter of which often exhibit performance deficiencies in low-permeability or complex reservoirs. Due to its capability for fine-scale nesting and matrix element refinement, the MINC model better captures microscopic flow characteristics, thereby enhancing simulation accuracy and reliability while facilitating reservoir management and optimization.

To account for the fluid flow between matrix blocks, Pruess et al. [53] proposed the dual-porosity dual-permeability (DPDP) model. Although the aforementioned models are fundamentally

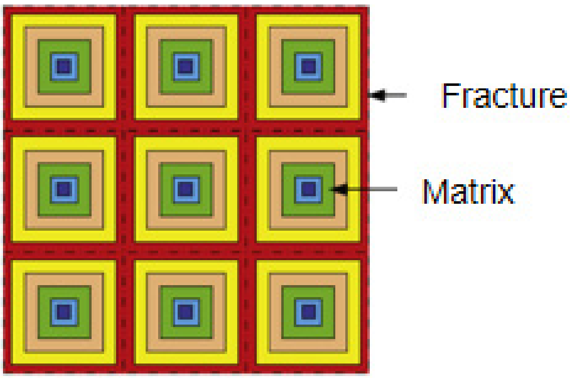

Fig.6 Conceptual MINC model (Wu and Pruess[52])

similar in terms of seepage laws, the Warren-Root dual-porosity model has become the most widely utilized conceptual dual-medium model in fractured reservoir simulation, owing to its simplistic principles, minimal input data requirements, and high computational efficiency and precision. Consequently, this model has been integrated into nearly all commercial oil and gas numerical simulators.

3.3 Discrete Fracture Model, DFM

DFM based on matching meshes generates unstructured grids to conform to the geometry of large-scale fractures, placing fractures on the interfaces of adjacent grid cells [54]. Various DFM schemes based on different numerical methods have been proposed to simulate single-phase and multiphase flow in fractured media: (1) Galerkin Finite Element Method (FEM): Noorishad and Mehran [55] employed a variational-based Galerkin FEM to simulate solute dispersion and convection. Karimi-Fard and Firoozabadi [56] proposed an FEM considering gravity and capillary forces in two-phase flow. However, for multiphase problems, the discrepancy in relative permeability curves between the matrix and fractures leads to inconsistent saturation at shared nodes, violating local mass conservation as Galerkin FEM requires identical degrees of freedom for both media at these nodes. (2) Block-Centered Finite Volume Method (FVM): FVM derives discrete schemes via integration and the divergence theorem. Karimi-Fard et al. [57] coupled two-phase control equations using FVM. Due to its clear physical significance and local mass conservation, FVM is widely integrated into commercial simulators. (3) Control Volume Finite Element Method (cvFEM): Compared to FEM, cvFEM satisfies local mass conservation. It effectively describes saturation discontinuities at matrix-fracture interfaces by assuming capillary force continuity, making it highly suitable for multiphase simulation while avoiding the complexity of FVM in handling specific fluxes [58]. (4) Mixed Finite Element Method (MFEM): This unstructured technique utilizes approximate expressions for edge fluxes, grid-center pressures, and edge pressures. By combining flux and pressure continuity, it eliminates flux terms to form a closed system, reducing degrees of freedom and computational costs [59]. (5) Boundary Element-based Methods: This technique uses local boundary integral equations to approximate mass transfer between the matrix and fractures. Fang [60] coupled this into an FEM framework for a global solution, enhancing single-phase accuracy but facing challenges in multiphase applications.

3.4 (Projected) Embedded Discrete Fracture Model

To circumvent the challenges associated with generating high-quality conforming unstructured meshes in the aforementioned Discrete Fracture Model (DFM), the Embedded Discrete Fracture Model (EDFM) was developed. This approach employs structured grid partitioning for the reservoir matrix and embeds fractures into these structured cells, explicitly characterizing them as source and sink terms. This method effectively integrates the flow characteristics of both the matrix and fractures, facilitating a more accurate simulation of fluid flow behavior in the reservoir. Li and Lee [61] first proposed the EDFM and utilized the Finite Volume Method (FVM), which satisfies local mass conservation, to simulate multiphase flow equations. Moinfar [62] established the first EDFM applicable to 3D fractured compositional reservoir models and analyzed the impact of natural and hydraulic fracture dynamic behaviors on the seepage field and horizontal well productivity. Wu et al. [63] processed Laplace-transformed single-phase flow equations using the Green's element method based on boundary integral equations, achieving high-precision calculations of early-stage flow regimes via Stehfest numerical inversion; however, this method is limited to single-phase flow. Rao et al. [64] refined the calculation formulae for inter-grid transmissibility and established connections between fracture grids projected onto the same matrix interface, eliminating significant errors present in the original pEDFM and EDFM for certain reservoir seepage problems. To date, the EDFM family (comprising EDFM and pEDFM) has been extensively applied in the numerical simulation of fractured reservoirs [65-72]. For instance, Ren [73, 74] proposed coupling the Extended Finite Element Method (XFEM) with pEDFM to study multiphase seepage in dynamic fractures. Yan et al. [75] enhanced the simulation precision of natural and induced fracture dynamics by incorporating the MINC model into an XFEM-EDFM framework. While the EDFM family avoids the difficulties of generating matching meshes, FVM-based EDFM and commercial simulators often struggle to efficiently and effectively characterize complex boundary geometries and conditions, such as Dirichlet boundaries with constant pressure or non-closed Neumann boundary conditions.

Regarding computational efficiency in reservoir simulation, two primary solutions exist: developing simplified numerical models or enhancing numerical calculation efficiency. Multi-scale methods are commonly employed to boost efficiency. Typical approaches include Multi-scale Finite Element Method (MsFEM) [76, 77], Multi-scale Mixed Finite Element Method (MsMFEM) [78-80], and Multi-scale Finite Volume Method (MsFVM) [81, 82]. Among these, MsMFEM and MsFVM satisfy local mass conservation, an essential feature for reservoir simulation. In recent years, scholars have applied multi-scale methods to fractured porous media. Zhang [83] proposed an MsMFEM for oil-water two-phase flow in fractured reservoirs. Zhang [84] introduced a DFM based on the multi-scale locally conservative Galerkin method, further improving the computational efficiency of MFEM-based models. Hajibeygi et al. [85] and Matei et al. [86] proposed an iterative multi-scale finite volume method combined with hierarchical fracture models. While these multi-scale methods have yielded satisfactory results and contributed significantly to simulating multiphase flow in fractured porous media, the construction of their basis functions remains complex. Furthermore, in practical field applications, they are constrained by memory overhead due to the limitations of grid systems. Given that both conforming mesh-based DFM and structured grid-based EDFM encounter issues in characterizing reservoir domains, this study is inspired to replace current grid-based methods with meshless methods. In meshless methods, the computational domain is discretized using nodes, which—unlike traditional grid discretization—is not restricted by grid topology. This allows for more flexible and free discrete representation of complex geometries, enabling the construction of simpler numerical models with fewer degrees of freedom. Consequently, meshless methods hold immense potential for applications in fractured reservoir numerical simulation.

3.5 Discrete Fracture-Vug Network Model

Fracture-vuggy reservoirs are distinguished by their pronounced heterogeneity and multi-scale pore structures. These reservoirs exhibit complex pore systems ranging from microscopic fractures to macroscopic vugs, resulting in highly intricate fluid flow regimes and uneven distribution of oil, gas, and water. The interconnectedness of fracture and vug systems constitutes a complex network, where flow within fractures obeys Darcy's law, whereas flow within vugs is characterized as free flow.

The coupling of these flow regimes, compounded by the complexities of multiphase flow, gravity segregation, and capillary forces, renders the simulation and prediction of fluid behavior extremely challenging [87-89]. In terms of development, fracture-vuggy reservoirs face issues of rapid production decline and low recovery factors, necessitating advanced technologies such as geological modeling, gas/water injection, and acid fracturing to enhance reserve mobilization and ultimate recovery [90, 91].

To simulate these complex flows more accurately, researchers have developed the Discrete Fracture-Vug Network (DFVN) model. In 2010, Yao et al. [92] first proposed an innovative DFVN model encompassing matrix, fracture, and vug systems; this model integrated vug and fracture systems while incorporating small-scale fractures and dissolution pores into the matrix system. In the same year, Huang et al. [93] introduced a macroscopic flow mathematical model for fracture-vuggy media, which partitioned the media into rock blocks, fractures, and vug systems. Based on homogenization theory, they performed multi-scale upscaling and derived theoretical solutions for equivalent Darcy flow equations and permeability tensors. In 2015, Zhang et al. [94] proposed a discrete numerical simulation method for fracture-vug units based on the statistical characteristics of physical parameters and effective parameters such as the fracture connectivity flow coefficient. In 2019, Zhang et al. [95] introduced a simulation method considering multi-scale fractures, utilizing an embedded approach for large-scale fractures and equivalent models for small-scale fractures and vugs.

In 2022, Yao and Zhang et al. [96] investigated coupled flow and heat transfer in fracture-vuggy karstic geothermal reservoirs, introducing the excluded volume concept from seepage theory and proposing an analytical expression for DFVN connectivity. They solved the coupled Navier-Stokes (free flow) and Darcy (porous media) equations to numerically derive the permeability of fractured karstic porous media and explored the relationship between connectivity and permeability.

Yan [97] proposed a hybrid model combining DFVN and Embedded Discrete Fracture Model (EDFM), utilizing oversampling techniques, volume averaging, and mimetic finite difference methods to simulate flow in reservoirs with large-scale conductive fractures. Liu et al. [98] developed an advanced numerical method coupling the Finite Volume Method (FVM) with an embedded discrete fracture-vug model to study condensate gas flow and production dynamics. Kou et al. [99] investigated the generation and propagation of wormholes in carbonate reservoirs based on Darcy-Forchheimer equations, using Mixed Finite Element Methods (MFEM) for solution. Yuan et al. [100] proposed a 3D acid-rock reaction model based on Stokes-Brinkman equations, finding that rock heterogeneity and mineral volume fraction significantly impact connectivity.

The DFVN model is an effective extension of the Discrete Fracture Network (DFN) approach, better reflecting the characteristics of fracture-vuggy reservoirs by incorporating vug systems [101-103]. While DFVN offers high accuracy by precisely describing large-scale features, it is constrained by low computational efficiency and difficulties in grid generation. Achieving a balance between high precision, computational efficiency, and simplified meshing remains a critical area for optimization, with profound implications for industrial applications.

3.6 Meshless Numerical Simulation

Meshless Methods, also referred to as the Element-Free Galerkin (EFG) method, represent a numerical simulation technology tailored for engineering and scientific problems. Distinguishable from traditional grid-based numerical methods (such as the Finite Element Method, FEM), meshless methods circumvent the need for domain meshing by directly employing a set of discrete nodes to construct approximation functions, thereby eliminating issues related to grid generation and mesh distortion. Key steps of meshless methods include:

(1) Point cloud discretization: Representing the computational domain with a set of discrete nodes; (2) Shape function construction: Constructing a basis function (also known as a shape or weight function) for each node, which defines a localized area of influence known as the support domain (or influence domain); (3) Field variable approximation: Utilizing basis functions to approximate continuous field variables (e.g., displacement, velocity, or pressure); (4) Governing equation construction: Transforming the problem into a set of algebraic equations based on physical laws and boundary conditions through mathematical tools like weighted residuals or variational principles. These equations typically take an integral form based on weighted averaging; however, due to the absence of a grid, specialized numerical integration methods are required; (5) System solving: Solving the resulting algebraic equations to obtain physical quantities at each node; (6) Post-processing: Extracting information such as stress distribution and displacement fields.

Early research into meshless methods dates back to the 1970s, encompassing the Vortex method [104, 105], irregular-grid finite difference methods or Generalized Finite Difference Method (GFDM) [106-108], and Smoothed Particle Hydrodynamics (SPH) [109, 110]. The core of meshless methods lies in utilizing a point cloud—a set of nodes without explicit connectivity—to approximate physical problems in a continuous domain, rather than relying on traditional finite element meshes. Point cloud generation involves random, uniform, geometry-based, or adaptive generation, often requiring iterations to ensure quality. By adopting point-based approximation, these methods offer significant flexibility and adaptability, independent of initial grid partitioning or reconstruction, ensuring computational precision while reducing complexity. Subsequently, Nayroles et al. [111] (1992) introduced Moving Least Squares (MLS) approximation into the Galerkin framework, proposing the Diffuse Element Method (DEM). Belytschko et al. [112] refined DEM into the Element-Free Galerkin (EFG) method, which exhibits higher precision and convergence rates than FEM without volume locking, albeit with higher computational costs and a reliance on background cells for numerical integration. In 1996, Spanish scholars Oñate and Idelsohn [113] proposed the Finite Point Method (FPM), which utilizes MLS for shape functions and a collocation scheme for discretization; as a truly meshless method requiring no background cells, it is primarily applied in fluid mechanics. In 1998, Atluri et al. introduced the Local Boundary Integral Equation (LBIE) method [114-116] and the

Meshless Local Petrov-Galerkin (MLPG) method [117-120], both of which utilize MLS for field function approximation and establish meshless schemes through the local Petrov-Galerkin method without background integration cells. Galerkin-based meshless methods offer high precision but require significant computation for numerical integration. Conversely, collocation-based methods are more computationally efficient but may lag in stability and precision. Zhang et al. [121] addressed these issues by proposing least-squares collocation meshless methods and weighted least-squares versions [122, 123]. Li et al. [124] first incorporated Moving Weighted Least Squares (MWLS) into numerical well testing for complex fault-block reservoirs, achieving satisfactory precision and efficiency.

From the perspective of domain discretization, meshless methods are categorized into global and local (domain-type) methods. Global methods construct approximation functions across the entire domain using techniques like MLS, achieving smooth approximations but potentially sacrificing efficiency. Local meshless methods, including EFG, local least squares, and point interpolation methods, construct functions within the local support domain of each node. These offer advantages in efficiency, localized problem handling, and adaptive capabilities, making them particularly suitable for challenging issues like complex geometries, large deformations, and fracture propagation.

Generalized Finite Difference Method (GFDM) is a local meshless method [125] that represents the domain using nodes and partitions it into intersecting influence sub-domains. Within each sub-domain, partial derivatives of unknown functions with respect to spatial variables can be expressed in difference schemes via multivariate Taylor series expansion and weighted least squares. This effectively overcomes the grid dependence of traditional FDM. Benito et al. [126] proposed an adaptive GFDM tailored to specific problems and domains. Based on Taylor expansion within the support domain, GFDM obtains second-order accurate difference approximations, functioning as a natural multi-point flux approximation scheme with high precision. This method has seen extensive application in solving high-order PDEs [127], stress analysis [128], shallow water equations [129], and anisotropic reservoir problems [139].

Rao et al. [136, 144] introduced an upwind scheme into GFDM, solving coupled thermo-flow problems in porous media via a sequential coupling scheme and two-phase flow via a fully implicit nonlinear solver, demonstrating excellent performance. Subsequently, Rao et al. [145, 146] incorporated nodal control volumes into GFDM and derived the Extended Finite Volume Method (EFVM) satisfying local mass conservation. Compared to traditional FVM-based reservoir simulation, EFVM flexibly characterizes complex geometries through collocation without meshing and handles more intricate boundary conditions. Moreover, EFVM can directly utilize existing nonlinear solvers from FVM-based technologies, facilitating the development of various reservoir simulators and revealing the immense potential of EFVM in the field. However, when applied to large-scale field problems, this method still faces challenges of computational time and memory consumption similar to classic grid-based methods. Additionally, due to the lack of explicit connectivity between nodes, a systematic method for connectivity characterization—analogous to streamline simulation in grid systems—has yet to be established for nodal systems.

4 Interwell Connectivity Analysis Methods

4.1 Reservoir Engineering Methods

Determining reservoir connectivity is a critical component of oilfield development and management. Currently, a variety of technical means are employed for comprehensive evaluation, including seismic logging, tracer testing, well test analysis, and chemical analysis. As a vital static analysis method, well logging is primarily used to acquire fundamental formation information. Precise detection of target intervals can be achieved through technologies such as conventional logging, water injection profile logging, and DLS logging. These logging techniques efficiently determine geological parameters such as porosity and permeability to infer interwell connectivity. Due to the inherent similarities in formations from the same depositional period, researchers can determine whether fluid communication exists between different wells through comparative analysis of logging curves.

For instance, Qian et al. [147] compared multiple representative logging curves to lay the foundation for subsequent refined reservoir studies; Wu [148] established mathematical models based on production logging data to invert interwell connectivity coefficients, further deepening the quantitative analysis of connectivity. Additionally, Wang et al. [149] successfully identified preferential flow paths in reservoirs by analyzing anomalous response characteristics in water injection profile logs, thereby improving the precision of geological interpretation. Meng et al. [150] adopted the Analytic Hierarchy Process (AHP) for the comprehensive evaluation of logging data, proposing a more efficient method for identifying preferential channels. Notably, Hong [151] introduced DLS logging curves and proposed a quantitative identification method to accurately characterize reservoir properties. Zhao [152] utilized carbon-oxygen ratio (C/O) spectral logging to calculate reservoir water saturation and evaluate the existence of large-scale channels. Shang [153] successfully controlled the issue of rising water cut in production wells using pulsed neutron oxygen activation logging data.

Among chemical analysis methods, crude oil chromatography fingerprinting is widely recognized as a classic tool for evaluating reservoir connectivity. Kaufman et al. [154] first proposed this method, which involves performing total hydrocarbon gas chromatography on oil samples collected from production wells to obtain fingerprint characteristics of oil samples from different layers. By comparing the similarity of these fingerprint features, interwell fluid connectivity can be effectively determined [155, 156]. Once a foundational chromatographic fingerprint database is established, oil samples can be accurately classified during the analysis process to identify key parameters for determining connectivity status. However, the high cost of chromatography fingerprinting limits its large-scale promotion and application in oilfields [157].

Furthermore, well test analysis reflects subsurface fluid flow by analyzing dynamic changes in pressure and production during the oilfield development process, thereby testing dynamic interwell connectivity [158]. Interference testing and pulse testing are common methods; by altering the operating mode of a specific well and monitoring the pressure response in other bottom-holes, connectivity can be assessed [159, 160]. Notably, pulse testing is extensively applied in medium-to-high permeability reservoirs. The new pulse testing theoretical charts proposed by Liu et al. [161] have been validated in field cases, making connectivity analysis more precise. Tiab et al.

[162] used bottom-hole pressure fluctuation data to quantitatively characterize the connectivity status between different wells.

Compared to interference testing, the pressure derivative curve analysis method can significantly save time [163]. Lin et al. [164] performed qualitative connectivity analysis by studying pressure recovery curves within the same well group through the comprehensive analysis of unsteady-state pressure test data. The high efficiency of well testing also makes it exceptional in identifying interwell preferential channels; Shi et al. [165] successfully identified such channels using pressure drawdown curves.

In summary, although logging, chemical analysis, and well testing each possess distinct characteristics and offer different perspectives on reservoir connectivity, they all have inherent limitations. Well logging, as a static analysis, is limited in its ability to capture dynamic reservoir changes; chemical analysis is constrained by high costs; and well testing faces challenges in widespread application due to its heavy workload and high testing expenses. While these methods can reveal interwell connectivity to a certain extent by effectively utilizing existing data, they are also subject to constraints such as simplified and idealized model assumptions, a lack of geologically meaningful inversion parameters, and the neglect of changing practical conditions, all of which impact the accuracy and reliability of model predictions.

4.2 Interwell Streamline Characterization Methods

The streamtube method originated as a pivotal tool for early geological modeling and reservoir numerical simulation. Its foundations trace back to 1937, when Muskat [166] first explored the definitions of potential functions and stream functions for incompressible fluids in two-dimensional space. In 1951, Fay and Pratts [167] initially applied streamtube models to simulate areal flow in homogeneous porous media. However, they observed that the actual positions of streamlines shift during the progression of multiphase displacement, yet they lacked effective methods to track such dynamic evolution.

In 1962, Higgins and Leighton [168] utilized streamtube bundles to simulate multiphase displacement processes. Within this framework, streamtubes were treated as one-dimensional systems, analyzed by solving the Buckley-Leverett equation along the tube's trajectory. While this method fixed the streamtube paths, it exhibited significant limitations in handling complex dynamic flows. In 1973, Martin [169] noted that traditional fixed-streamtube models were inadequate for two-phase flow problems with mobility ratios less than 1 or greater than 100. Consequently, a strategy involving periodic updates of streamtube paths was introduced to enhance simulation accuracy.

By 1979, Martin and Wegner [170] extended this methodology to two-phase flow simulation in multi-well systems, further enriching the applications of the streamtube method. Although streamtube methods offered superior computational speed and efficiency compared to traditional finite difference methods (FDM), they continued to face challenges during complex displacement processes, particularly in characterizing fluid heterogeneity and shifting flow behaviors.

Entering the 1990s, streamline models gradually superseded streamtube methods as the mainstream approach for reservoir simulation [171, 172]. In 1995, Datta-Gupta [173] introduced streamline research into the petroleum industry, proposing a numerical solution for the saturation equation along streamlines, which replaced the previous analytical solutions along streamtubes. This shift established the foundation for the flexibility and practicality of streamline simulation. This approach not only simplified model complexity but also broadened the research scope, enabling it to effectively address diverse geological conditions.

With the rapid advancement of computer technology, streamline modeling encountered new opportunities. Researchers began utilizing streamline models to perform rapid screening across various stochastic geological models to better reflect reservoir heterogeneity. Given the complexity of modern geological models—where fine-grid nodes can reach hundreds of thousands or even millions—the advantages of streamline models in rapid computation have become increasingly prominent. In 1996, Peddibhotla [174] from Texas A&M University compared a water-oil displacement streamline model with commercial software, finding that the new model far outperformed traditional methods in speed while maintaining comparable accuracy.

4.3 Interwell Connectivity Analysis Methods (INSIM Series)

In 2014, Zhao et al. [175] proposed a method to simplify reservoir injection-production systems by establishing an Interwell Numerical Simulation Model (INSIM) based on dynamic injection and production data. This model enables rapid inversion to obtain interwell formation parameters and provides real-time visualization of interwell connectivity. Building upon this, Zhao et al. [176] (2015) introduced a physics-based data-driven model also termed INSIM, capable of rapidly predicting the dynamic behavior of oil-water two-phase flow [177]. In 2016, Zhao et al. [178] incorporated the influence of aquifers and explored the model's potential for production optimization. That same year, Zhao et al. [179] conducted layered connectivity analysis and established an inversion model for interwell connectivity in multi-layer waterflooded reservoirs.

In 2017, Xie et al. [180] extended the INSIM framework to polymer flooding, developing a connectivity-based method to predict polymer concentration and key indicators such as polymer breakthrough time. Subsequently, Guo et al. [181] (2018) proposed the improved INSIM-FT model, which incorporates a front-tracking (FT) method to solve the Buckley-Leverett equation, achieving more accurate water-cut predictions and facilitating production optimization studies [182]. In 2019, Zhao et al. [183] developed a method based on INSIM to evaluate injection efficiency and rapidly optimize injection-production rates. Concurrently, Guo et al. [184] established a corresponding 3D model to account for the effects of reservoir gravity.

In 2020, Zhao et al. [185] enhanced the functionality of the INSIM model by introducing infill nodes and a flow-path tracking (FPT) algorithm, resulting in the INSIM-FPT model. This advancement allowed for the calculation of interwell flow paths and dynamic allocation factors (split coefficients). By 2021, Liu et al. [186] proposed a novel optimization method for injection-production rates based on INSIM-FPT, with a focus on oil-cut evaluation. Simultaneously, Zhao et al. [187] expanded the application scope by constructing the INSIM-FPT-3D model, which considers gravity and utilizes a new productivity index (well index) calculation method to improve the accuracy of bottom-hole flowing pressure (BHFP) calculations. Additionally, Liu et al. [188] utilized these models for dynamic prediction of profile control and water plugging, demonstrating that the model effectively reflects reservoir dynamic changes following such treatments.

In recent years, the evolution and application of INSIM-type models have demonstrated immense potential in reservoir management and connectivity assessment. Their primary characteristics include: the ability to perform dynamic calculations and predictions for oil-water two-phase flow; the use of dynamic allocation factors to reflect changes in the reservoir flow field; and the capacity to accurately account for the impact of shut-in and conversion wells, thereby effectively characterizing interwell connectivity.

5 Analysis and Discussion

(1) Reservoirs often harbor natural fractures, which induce anisotropy. Space transformation theory, commonly used to address anisotropy, lacks universality, while multi-point flux approximation (MPFA) involves complex computational procedures.

Space transformation theory converts anisotropic problems into isotropic ones via coordinate transformation. Although simple and efficient, it increases the complexity of boundary conditions. Two-point flux approximation (TPFA), based on finite difference schemes, approximates interface flux using pressures from two adjacent grids to determine transmissibility. This method is limited to orthogonal grids and diagonal permeability tensors, failing to handle unstructured grids and full tensors. In contrast, MPFA is based on the control volume method under flux and pressure continuity conditions. It partitions neighboring grids into several interaction regions to approximate interface flux, making it suitable for unstructured grids and full permeability tensors. However, MPFA is computationally intensive and suffers from poor stability.

(2) Actual reservoir geological conditions are intricate, frequently containing complex media such as fractures, faults, and vugs. Traditional grid-based methods face difficulties in characterizing these media and suffer from low computational efficiency.

Current methods for characterizing complex media include grid refinement, which uses high-density meshes for geometric precision but faces massive computational loads and convergence issues. Multi-continuum models (e.g., dual-porosity models) treat the reservoir as multiple systems (matrix, fractures, vugs) with defined mass exchange. While suitable for uniformly distributed media, they fail to accurately depict the geometry of specific complex features. Discrete Fracture Models (DFM) match fracture geometry using unstructured grids, but generating high-quality conforming meshes for complex networks is challenging. Embedded Discrete Fracture Models (EDFM) embed fractures as source/sink terms in structured matrix grids; while better at characterizing fracture shapes, they struggle with complex boundary geometries. To balance precision and efficiency, multi-scale methods have been proposed. The core idea is to partition grids at a macroscopic scale and solve local differential equations to construct multi-scale basis functions that capture small-scale flow features. However, multi-scale methods demand high-quality meshing; poor partitioning leads to ill-conditioned linear systems and non-convergence. Furthermore, their universality across different types of complex media remains limited.

(3) Existing grid-based and meshless methods cannot directly reveal the flow relationships in complex reservoirs or quantitatively characterize connectivity. Currently, streamline simulation is the most widely applied and theoretically mature method. Based on a background grid system, it utilizes streamline tracking to obtain interwell flow relationships. The most common approach, developed by Datta-Gupta based on Pollock's tracking method, solves the saturation equation along 1D

streamlines. While more accurate and faster than traditional methods, streamline tracking primarily reveals injection-production relationships and fails to recognize connectivity between production wells (or aquifers) or effectively handle complex media like fractures and vugs. Moreover, a systematic method analogous to streamline simulation for quantitative connectivity characterization in meshless systems has yet to be established.

Complex networks have become a prominent research topic, focusing on topology, dynamic models, and commonalities across networks to reveal inter-object relationships. Networks consist of nodes and edges, with the node degree defined as the number of adjacent edges. Classic models include random networks, small-world models, and scale-free networks, primarily characterized by their degree distribution. Recently, a physical network model was proposed in reservoir simulation—a data-driven framework that discretizes interwell spaces into 2D grid topologies to invert seepage properties using historical data. While complex networks offer new perspectives for connectivity characterization, their current dynamic behaviors are limited to diffusion and random walks, lacking the rigorous physical significance required for reservoir flow simulation.

In summary, current reservoir numerical simulation faces three major challenges: 1. Reservoir anisotropy, where space transformation is not universal and MPFA is complex and unstable; 2. Difficult characterization and low efficiency for fractures, faults, and vugs; 3. The inability to directly reveal and quantitatively characterize dynamic flow relationships between any wells. Therefore, it is imperative to research meshless numerical simulation methods that can flexibly characterize complex geometries with high stability and efficiency, and to develop technologies for the quantitative characterization of dynamic connectivity to support the efficient development of complex reservoirs.

6 Conclusion

The review and analysis presented in this paper indicate that although numerical simulation and connectivity analysis technologies for complex fractured-vuggy reservoirs have made significant progress, core challenges remain.

First, in addressing anisotropy, the existing space transformation theory lacks universality, while the multi-point flux approximation (MPFA) methods are computationally complex and suffer from poor stability.

Second, regarding complex media such as fractures and vugs, traditional grid-based methods face difficulties in geometric characterization and exhibit low computational efficiency. Specifically, equivalent continuum models struggle to reflect true flow regimes; Discrete Fracture Models (DFM) involve complex mesh generation; and Embedded Discrete Fracture Models (EDFM) perform poorly in handling complex boundary conditions.

Third, current methods within both grid-based and meshless frameworks struggle to directly and quantitatively reveal the dynamic flow relationships between arbitrary wells. Although streamline simulation can characterize injection-production relationships, it faces challenges in handling complex media and fails to identify connectivity between production wells (or aquifers).

Consequently, future research must prioritize the development of a new generation of reservoir numerical simulation methods that can flexibly characterize complex geometries with high stability, efficiency, and broad applicability. Furthermore, it is essential to explore new technologies for the effective quantitative characterization of dynamic connectivity. This includes developing novel connectivity analysis models based on meshless systems that integrate both physical significance and data-driven characteristics, providing robust technical support for the efficient development and decision-making of complex reservoirs.

Acknowledgement

None.

Funding Statement

None.

Author Contributions

The author confirms sole responsibility for the following: study conception and design, data-collection, analysis and interpretation of results, and manuscript preparation.

Availability of Data and Materials

None.

Conflicts of Interest

The authors declare that they have no conflicts of interest to report regarding the present study.

References

[1] Jia Chengzao, Wang Zugang, Jiang Lin, et al. Progress and scientific and technological issues of shale oil exploration and development in China [J]. World Petroleum Industry, 2024, 31(04): 1-11+13.

[2] Luo Pingya, Zhu Suyang. Theoretical and technical foundation for establishing a large-scale coalbed methane industry with 100 billion cubic meters in China [J]. Acta Petrolei Sinica, 2023, 44(11): 1755-1763.

[3] Liu He. Application of artificial intelligence in digital transformation of oil and gas exploration and development is the trend of the times [J]. Oil Forum, 2023, 42(03): 1-9+47.

[4] Peng Kai, Fang Duo, Wu Chigong. Generation of orthogonal numerical grid and calculation of two-dimensional flow field [J]. Journal of Hydraulic Engineering, 1990(4): 9.

[5] Xiao Hanshan, Chen Zuobin, Luo Gang, et al. Application of adaptive Cartesian grid in numerical simulation of flow field around 3D complex shapes [C]// Proceedings of the 4th Cross-Strait Symposium on Computational Fluid Dynamics. 2003: 9.

[6] Hou Xiaolin, Yang Bo. Surface model extraction algorithm of 3D geological bodies based on pillar grid [J]. Computer Engineering and Applications, 2017, 53(11): 1-6.

[7] Wang Daigang, Hou Jian, Xing Xuejun, et al. An improved PEBI grid generation method based on front-advancing [J]. Chinese Journal of Computational Physics, 2012, 29(5): 9.

[8] Xie Haibing, Huan Guanren, Guo Shangping, et al. Two-dimensional two-phase flow numerical simulation with PEBI grid [J]. Acta Petrolei Sinica, 1999, (02): 65-69+6.

[9] Tian Junwei, Cheng Gang. Improved Delaunay triangulation algorithm [J]. Journal of Xi'an Technological University, 2011, 31(4): 6.

[10] Shi Yijing, Shao Guojian, Ding Shengyong. Generation method of polygonal element grid based on Voronoi structure [J]. Journal of Henan University of Science and Technology (Natural Science), 2016, 37(05): 51-55+6-7.

[11] He J. Numerical Simulation of a Class I Gas Hydrate Reservoir Depressurized by a Fishbone Well[J]. Processes 2023, 11, 771.

[12] Lei Q, Doonechaly N G, Tsang C F. Modelling fluid injection-induced fracture activation, damage growth, seismicity occurrence and connectivity change in naturally fractured rocks[J]. International Journal of Rock Mechanics and Mining Sciences, 2021, 138: 104598.

[13] Russell D, Wang Z J. A Cartesian grid method for modeling multiple moving objects in 2D incompressible viscous flow[J]. Journal of Computational Physics, 2003, 191(1): 177-205.

[14] Eldredge J D. A method of immersed layers on Cartesian grids, with application to incompressible flows[J]. Journal of Computational Physics, 2022, 448: 110716.

[15] Zhao P, Xu J, Liu X, et al. A computational fluid dynamics-discrete element-immersed boundary method for Cartesian grid simulation of heat transfer in compressible gas–solid flow with complex geometries[J]. Physics of Fluids, 2020, 32(10).

[16] Ponting D K. Corner point geometry in reservoir simulation[C]//ECMOR I-1st European conference on the mathematics of oil recovery. European Association of Geoscientists & Engineers, 1989: cp-234-00003.

[17] Andersen O, Nilsen H M I I, Raynaud X. Coupled geomechanics and flow simulation on corner-point and polyhedral grids[C]//SPE Reservoir Simulation Conference[J]. SPE, 2017: D031S010R004.

[18] Xu Y, Fernandes B R B, Marcondes F, et al. Embedded discrete fracture modeling for compositional reservoir simulation using corner-point grids[J]. Journal of Petroleum Science and Engineering, 2019, 177: 41-52.

[19] Mao Xiaoping, Zhang Zhiting, Qian Zhen. Analysis of geological models expressed by corner-point grid and its application in the simulation of hydrocarbon accumulation process [J]. Journal of Geology, 2012(03): 45-53.

[20] Li Muzhe, Hou Xiaolin, Yang Bo, et al. Surface extraction algorithm of complex geological models based on corner-point grid [J]. Computer Engineering and Applications, 2016, 52(9): 6.

[21] Lin Chunyang. Research, development and application of reservoir numerical simulator based on PEBI grid [D]. University of Science and Technology of China, 2010.

[22] Heinemann Z E. Interactive generation of irregular simulation grids and its practical applications[C] SPE University of Tulsa Centennial Petroleum Engineering Symposium. SPE, 1994: SPE-27998-MS.

[23] Liu G R, Gu Y T. An introduction to meshfree methods and their programming[M]. Springer Science & Business Media, 2005.

[24] Zhao Hui, Liu Wei, Rao Xiang, et al. Connection element method for reservoir numerical simulation [J]. Scientia Sinica Technologica, 2022, 52(12): 1869-1886.

[25] Zhao H, Zhan W, Yuhui Z, et al. A connection element method: Both a new computational method and a physical data-driven framework—-Take subsurface two-phase flow as an example[J]. Engineering Analysis with Boundary Elements, 2023, 151: 473-489.

[26] Snow D T. Rock fracture spacings, openings and porosities[J]. Journal of the Soil Mechanics & Foundations Division, 1968, 94(1):73-92.

[27] Oda, Mc. “Permeability tensor for discontinuous rock masses[J].” Geotechnique 35. 4 (1985): 483-495.

[28] Lough M F, S H Lee, and J Kamath. An efficient boundary integral formulation for flow through fractured porous media[J]. Journal of Computational Physics 143. 2 (1998): 462-483.

[29] Lee S H, C L Jensen, and M F Lough. An efficient finite difference model for flow in a reservoir with multiple length-scale fractures[J]. SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers, 1999.

[30] Fang Sidong. Research on numerical simulation of fractured horizontal wells using hybrid boundary element method [D]. China University of Petroleum (Beijing), 2017.

[31] Muskat M, Meres M W. The flow of heterogeneous fluids through porous media[J]. Physics, 1936, 7(9): 346-363.

[32] An Yongsheng. Analysis of the impact of reservoir anisotropy on the productivity of fishbone multilateral wells [J]. Journal of Southwest Petroleum University (Science & Technology Edition), 2011,

33(03): 145-148+201.

[33] Møyner O, Lie K A. A multiscale two-point flux-approximation method[J]. Journal of Computational Physics, 2014, 275: 273-293.

[34] Rao X, He X, Du K, et al. A novel projection-based embedded discrete fracture model (pEDFM) for anisotropic two-phase flow simulation using hybrid of two-point flux approximation and mimetic finite difference (TPFA-MFD) methods[J]. Journal of Computational Physics, 2024, 499: 112736.

[35] Sandve T H, Berre I, Nordbotten J M. An efficient multi-point flux approximation method for discrete fracture–matrix simulations[J]. Journal of Computational Physics, 2012, 231(9): 3784-3800.

[36] Chen Q Y, Wan J, Yang Y, et al. A new multipoint flux approximation for reservoir simulation[C]// SPE Reservoir Simulation Conference. SPE, 2007: SPE-106464-MS.

[37] Aavatsmark I. An introduction to multipoint flux approximations for quadrilateral grids[J]. Computational Geosciences, 2002, 6: 405-432.

[38] Contreras F R L, Lyra P R M, Souza M R A, et al. A cell-centered multipoint flux approximation method with a diamond stencil coupled with a higher order finite volume method for the simulation of oil–water displacements in heterogeneous and anisotropic petroleum reservoirs[J]. Computers & Fluids, 2016, 127: 1-16.

[39] Chen Q Y, Wan J, Yang Y, et al. Enriched multi-point flux approximation for general grids[J]. Journal of Computational Physics, 2008, 227(3): 1701-1721.

[40] Barenblatt G I, Zheltov I P, Kochina I N. Basic concepts in the theory of seepage of homogeneous liquids in fissured rocks[J]. Journal of Applied Mathematics & Mechanics, 1960, 24(5):1286-1303.

[41] Warren J E, Root P J. The Behavior of Naturally Fractured Reservoirs[J]. Society of Petroleum Engineers Journal, 1963, 3(3):245-255.

[42] Coats K H. Implicit compositional simulation of single-porosity and dual-porosity reservoirs[J]. SPE 18427, SPE Reservoir Simulation Symposium, Houston, TX, 1989, 6-8.

[43] Lim K T, Aziz K. Matrix-fracture transfer shape factors for dual porosity simulators[J]. Journal of Petroleum Science and Engineering, 1995, Vol. 13, 169-178.

[44] Kazemi H. Seth M S, Thomas G W. Pressure Transient Analysis of Naturally Fractured Reservoirs with Uniform Fracture Distribution[J]. Society of Petroleum Engineers Journal, 1969, 9(4):451-462.

[45] James R Glman, Hossein Kazemi. Improvements in simulation of naturally fractured Reservoirs[C]// SPE 10511. 1982.

[46] Deswaan O A. Analytic solutions for determining naturally fractured reservoir properties by well testing[C]// SPE 5346. 1976.

[47] Cigun L. The unsteady radial flow of compressible liquids through a medium with multiple porosity[C]//SPE International Oil and Gas Conference and Exhibition in China. SPE, 1982: SPE-10580-MS.

[48] Abdassah D, Ershaghi I. Triple- porosity systems for representing naturally fractured reservoirs[J]. SPE 13409. 1986.

[49] Wu Y S, Moridis G, Berkeley L, et al. A multicontinuum model for gas production in tight fractured reservoirs[C]//SPE 118944. SPE Hydraulic Fracturing Technology Conference. Society of Petroleum Engineers. 2009.

[50] Pruess K. GMINC- A mesh generator for flow simulations in fractured reservoirs[J]. Lawrence Berkeley Laboratory. 2008.

[51] Gilman J R, Kazemi H. Improvements in simulation of naturally fractured reservoirs[J]. Society of Petroleum Engineers Journal, 1982, 23:4(4):695-707.

[52] Wu Y S, Pruess K. A Multiple-Porosity Method for Simulation of Naturally Fractured Petroleum Reservoirs[J]. Spe Reservoir Engineering, 1988, 3(1):327-336.

[53] Pruess K. TOUGH2: A general-purpose numerical simulator for multiphase fluid and heat flow[J]. Nasa Sti/recon Technical Report N, 1991, 92.

[54] Noorishad J, Mehran M. An upstream finite element method for solution of transient transport equation in fractured porous media[J]. Water Resources Research, 1982, 18(3):588-596.

[55] Karimi-Fard M, Firoozabadi A. Numerical Simulation of Water Injection in 2D Fractured Media Using Discrete-Fracture Model[J]. SPE Reservoir Evaluation & Engineering, 2001, 4.

[56] Karimi-Fard M, Durlofsky L J, Aziz K. An Efficient Discrete-Fracture Model Applicable for General-Purpose Reservoir Simulators[J]. SPE Journal, 2004, 9(2): 227-236.

[57] Monteagudo J E P, Firoozabadi A. Control-volume method for numerical simulation of two-phase immiscible flow in two and three dimensional discrete-fractured media[J]. Water Resources Research, 2004, 40(7):7405.

[58] Matthi S K, Mezentsev A, Belayneh M. Control-Volume Finite-Element Two-Phase Flow Experiments with Fractured Rock Represented by Unstructured 3D Hybrid Meshes[C]// SPE Reservoir Simulation Symposium. Society of Petroleum Engineers, 2005.

[59] Chavent G, Roberts J. A unified physical presentation of mixed, mixed-hybrid finite element method and standard finite difference approximations for the determination of velocities in water flow problems[J]. Advances in Water Resources, 1991, 14(6): 329–33.

[60] Fang S, Cheng L, Ayala L F. A coupled boundary element and finite element method for the analysis of flow through fractured porous media[J]. Journal of Petroleum Science and Engineering, 2017, 152: 375-390.

[61] Lee S H, Jensen C L, Lough M F. Efficient finite-difference model for flow in a reservoir with multiple length-scale fractures[J]. SPE Journal, 2000, 5(03): 268-275.

[62] Moinfar A, Varavei A, Sepehrnoori K, et al. Development of an efficient embedded discrete fracture model for 3D compositional reservoir simulation in fractured reservoirs[J]. SPE Journal, 2014, 19(02): 289-303.

[63] Wu Y, Cheng L, Fang S, et al. A Green element method-based discrete fracture model for simulation of the transient flow in heterogeneous fractured porous media[J]. Advances in Water Resources, 2020, 136:103489.

[64] Rao X, Cheng L, Cao R, et al. An efficient three-dimensional embedded discrete fracture model for production simulation of multi-stage fractured horizontal well[J]. Engineering Analysis with Boundary Elements, 2019, 106: 473-492.

[65] Ţene M, Bosma S B M, Al Kobaisi M S, et al. Projection-based embedded discrete fracture model (pEDFM)[J]. Advances in Water Resources, 2017, 105: 205-216.

[66] Jiang J, Younis R M. An improved projection-based embedded discrete fracture model (pEDFM) for multiphase flow in fractured reservoirs[J]. Advances in water resources, 2017, 109: 267-289.

[67] Rao X, Cheng L, Cao R, et al. A modified projection-based embedded discrete fracture model (pEDFM) for practical and accurate numerical simulation of fractured reservoir[J]. Journal of Petroleum Science and Engineering, 2020, 187: 106852.

[68] Li J, Yu W, Guerra D, et al. Modeling wettability alteration effect on well performance in Permian basin with complex fracture networks[J]. Fuel, 2018, 224: 740-751.

[69] Dachanuwattana S, Jin J, Zuloaga-Molero P, et al. Application of proxy-based MCMC and EDFM to history match a Vaca Muerta shale oil well[J]. Fuel, 2018, 220: 490-502.

[70] Wang Y, Shahvali M. Discrete fracture modeling using Centroidal Voronoi grid for simulation of shale gas plays with coupled nonlinear physics[J]. Fuel, 2016, 163: 65-73.

[71] Shakiba M, de Araujo Cavalcante Filho J S, Sepehrnoori K. Using Embedded Discrete Fracture Model (EDFM) in numerical simulation of complex hydraulic fracture networks calibrated by microseismic monitoring data[J]. Journal of Natural Gas Science and Engineering, 2018, 55: 495-507.

[72] Rao X, Cheng L, Cao R, et al. A modified em-

bedded discrete fracture model to study the water blockage effect on water huff-n-puff process of tight oil reservoirs[J]. Journal of Petroleum Science and Engineering, 2019, 181: 106232.

[73] Ren G, Jiang J, Younis R M. A Model for Coupled Geomechanics and Multiphase Flow in Fractured Porous Media Using Embedded Meshes[J]. Advances in Water Resources, 2018, 122: 113-130.

[74] Ren G, Rami Y. An integrated numerical model for coupled poro-hydro-mechanics and fracture propagation using embedded meshes[J]. Computer Methods in Applied Mechanics and Engineering, 2021, 376(1): 113606.

[75] Yan X, Huang Z Q, Yao J, et al. An efficient numerical hybrid model for multiphase flow in deformable fractured-shale reservoirs[J]. SPE Journal, 2018, 23(04): 1412-1437.

[76] Hou T Y, Wu X H. A multiscale finite element method for elliptic problems in composite materials and porous media[J]. Journal of computational physics, 1997, 134(1): 169-189.

[77] Efendiev Y R, Hou T Y, Wu X H. Convergence of a nonconforming multiscale finite element method[J], SIAM Journal on Numerical Analysis. 2000, 37(3): 888–910.

[78] Zhang N, Yan B, Sun Q, et al. Improving multiscale mixed finite element method for flow simulation in highly heterogeneous reservoir using adaptivity[J]. Journal of Petroleum Science and Engineering, 2017, 154: 382-388.

[79] Aarnes J E , Krogstad S, Lie K A. A hierarchical multiscale method for two-phase flow based upon mixed finite elements and nonuniform coarse grids, Multiscale Model[J]. Multiscale Modeling & Simulation, 2006, 5(2): 337–363.

[80] Zhang Q, Huang Z, Yao J. et al. Multiscale Mimetic Method for Two-Phase Flow in Fractured Media Using Embedded Discrete Fracture Model[J]. Advances in Water Resources, 2017, 107: 180–190.

[81] Hughes T J, Feijóo G R, Mazzei L, et al. The variational multiscale method—a paradigm for computational mechanics[J]. Computer methods in applied mechanics and engineering, 1998 166 (1) : 3–24.

[82] Jenny P, Lee S, Tchelepi H. Multi-scale finite-volume method for elliptic problems in subsurface flow simulation[J], Journal of Computational Physics, 2003, 187 (1): 47–67.

[83] Zhang Q, Huang Z, Yao J. et al. A Multiscale Mixed Finite Element Method with Oversampling for Modeling Flow in Fractured Reservoirs Using Discrete Fracture Model[J]. Journal of Computational and Applied Mathematics, 2017, 323: 95–110.

[84] Zhang N, Sun Q, Wang Y T, et al. A Locally Conservative Multiscale Finite Element Method for Multiphase Flow Simulation through Heterogeneous and Fractured Porous Media[J]. Journal of Computational and Applied Mathematics. 343 (2018): 501-519.

[85] Hajibeygi H, Karvounis D, Jenny P. A hierarchical fracture model for the iterative multiscale finite volume method[J]. Journal of Computational Physics, 2011, 230(24): 8729-8743.

[86] Ţene M, Al Kobaisi M S, Hajibeygi H. Algebraic multiscale method for flow in heterogeneous porous media with embedded discrete fractures (F-AMS) [J]. Journal of Computational Physics, 2016, 321: 819-845.

[87] Kang Zhijiang, Di Yuan, Zhao Yanyan, et al. Characteristics of fluid flow in vugs for fractured-vuggy reservoirs [J]. Petroleum Geology and Oilfield Development in Daqing, 2014, 33(03): 82-85.

[88] Cui Shuyue, Di Yuan. Numerical simulation of fractured-vuggy reservoirs based on gravity segregation assumption [J]. Journal of Basic Science and Engineering, 2020, 28(02): 331-341.

[89] Cheng Qian. Study on flow laws of fractured-vuggy carbonate reservoirs in Tahe Oilfield [D]. Graduate University of Chinese Academy of Sciences (Institute of Porous Flow Fluid Mechanics), 2009.

[90] Li Yang. Theory and method of development for carbonate fractured-vuggy reservoirs in Tahe Oilfield [J]. Acta Petrolei Sinica, 2013, 34(01): 115-121.

[91] Wang Yong. Study on oil-water and oil-gas two-

phase flow laws in fractured-vuggy carbonate reservoirs [D]. China University of Petroleum (Beijing), 2018.

[92] Yao Jun, Huang Zhaoqin, Wang Zisheng, et al. Discrete fracture-vug network model for flow in fractured-vuggy reservoirs [J]. Acta Petrolei Sinica, 2010, 31(05): 815-819+824.

[93] Huang Zhaoqin, Yao Jun, Li Yajun, et al. Permeability analysis of fractured-vuggy media based on homogenization theory [J]. Scientia Sinica Technologica, 2010, 40(09): 1095-1103.

[94] Zhang Hongfang. Research on discrete numerical simulation method for fracture-vug units in carbonate reservoirs [J]. Petroleum Drilling Techniques, 2015, 43(02): 71-77.

[95] Zhang Dongli, Cui Shuyue, Zhang Yun. Multi-scale fracture simulation method for fractured-vuggy reservoirs [J]. Chinese Journal of Hydrodynamics (Series A), 2019, 34(01): 13-20.

[96] Yao Jun, Zhang Xu, Huang Zhaoqin, et al. Coupled numerical simulation of fluid flow and heat transfer in fractured-vuggy karstic geothermal reservoirs [J]. Natural Gas Industry, 2022, 42(04): 107-116.

[97] Yan Xia, Huang Zhaoqin, Li Yang, et al. A hybrid model for fractured-vuggy reservoirs based on discrete fracture-vug network model [J]. Journal of Central South University (Science and Technology), 2017, 48(9): 2474-2483.

[98] Liu L, Fan W, Sun X, et al. Gas condensate well productivity in fractured vuggy carbonate reservoirs: A numerical modeling study[J]. Geoenergy Science and Engineering, (2023), 225, 211694.

[99] Kou J, Sun S, Wu Y. Mixed finite element-based fully conservative methods for simulating wormhole propagation[J]. Computer Methods in Applied Mechanics & Engineering, 2016, 298: 279-302.

[100] Yuan T, Ning Y, Qin G. Numerical Modeling and Simulation of Coupled Processes of Mineral Dissolution and Fluid Flow in Fractured Carbonate Formations[J]. Transport in Porous Media, 2016, 114(3): 747-775.

[101] Zhao Lei. Numerical simulation of discrete medium network model for fractured-vuggy reservoirs [D]. Southwest Petroleum University, 2013.

[102] Huang Zhaoqin. Theoretical research on multi-scale two-phase flow simulation based on discrete fracture-vug network model [D]. China University of Petroleum (East China), 2012.

[103] Li Longxin. Research on numerical simulation method of multi-scale carbonate fractured-vuggy reservoirs [D]. Southwest Petroleum University, 2013.

[104] Chorin A J, Numerical study of slightly viscous flow[J]. J. of Fluid Mechanics, 1973, 57, 785-796.

[105] Bernard P S. A deterministic vortex sheet method for boundary layer flow[J]. J. of Computational Physics, 1995, 117, 132-145.

[106] Girault V. Theory of a GDM on irregular networks[J]. SIAM journal on numerical analysis, 1974, 11: 260-282.

[107] Pavlin V and Perrone N, Finite difference energy rechniques for arbitrary meshes[J]. Comput Struct. 1975, 5: 45-58.

[108] Krok J, Orkisz J. A unified approach to the FE generalized variational FD method for nonlinear mechanics[J]. Concept and numerical approach. 1989, 353-362. Springer-Verlag.

[109] Lucy L. A Numerical Approach to Testing the Fission Hypothesis[J], Astron. J.1977, 82, 1013 - 1024.

[110] Gingold R A, Moraghan J J. Smooth particle hydrodynamics:theory and applications to non spherical stars[J]. Monthly notices of the royal astronomical society, 1977, 181, 375-389.

[111] Nayroles B, Touzot G, Villon P. Generalizing the finite element method: diffuse approximation and diffuse elements[J]. Computational mechanics, 1992, 10(5): 307-318.

[112] Belytschko T, Lu Y Y, Gu L. Element-free Galerkin methods[J]. International journal for numerical methods in engineering, 1994, 37(2): 229-256.

[113] Onate E, Idelsohn S, Zienkiewicz O C. et al. A finite point method in computational mechanics: Applications to convective transport and fluid flow[J].

International journal for numerical methods in engineering, 1996, 39: 3839~3866.

[114] Zhu T, Zhang J, Atluri S N. A local boundary integral equation(LBIE) method in computational mechanics, and a meshless discretization approach[J]. Computational mechanics, 1998, 21: 223~235.

[115] Zhu T, Zhang J, Atluri S N. A meshless local boundary integral equation (LBIE) method for solving nonlinear problems[J]. Computational mechanics, 1998, 22 : 174-186.

[116] Atluri S N, Sladek J, Sladek V et al. The Loacl boundary integral equation(LBIE) and it's meshless implementation for linear elasticity[J]. Computational mechanics, 2000, 25 :180~198.

[117] Atluri S N, Zhu T. A new meshless local Petrov-Galerkin(MLPG) approach in computational mechanics[J]. Computational mechanics, 1998, 22:117~127.

[118] Atluri S N, Zhu T. The meshless local Petrov-CGalerkin (MLPG) approach for solving problems in elasto-statics[J]. Computational mechanics, 2000, 25:169~179.

[119] Atluri S N, Cho J Y, Kim H G. Analysis of thin beams, using the meshless local Petrov- Galerkin method, with generalized moving least squares interpolations[J]. Computational mechanics. 1999, 24 : 334~347.

[120] Atluri S N, Kim H G, Cho J Y. A critical assessment of the truly Meshless Local Petrov-Galerkin (MLPG), and Local Boundary Integral Equation (LBIE) methods[J]. Computational mechanics, 1999, 24 : 348~372.

[121] Zhang X, Xiao-Hu Liu, Kang-Zu Song, et al. Least-squares collocation meshless method[J]. International Journal for Numerical Methods in Engineering, 2010, 51(9):1089-1100.

[122] Zhang Xiong, Hu Wei, Pan Xiaofei. Weighted least-squares meshless method [J]. Acta Mechanica Sinica, 2003, 35(4): 425-431.

[123] Yan L , Xiong Z , Mingwan L U . Meshless Least-Squares Method for Solving the Steady-State Heat Conduction Equation[J]. Tsinghua Science & Technology, 2005, 10(001):61-66.

[124] Li Yu-kun. Meshless Numerical Well-Test on Complex Fault-Block Reservoir[D]. China University of petroleum, 2007.

[125] Benito J J, Ureña F, Gavete L. Influence of several factors in the generalized finite difference method[J]. Applied Mathematical Modelling, 2001, 25(12): 1039–53.

[126] Benito J J, Ureña F, Gavete L, et al. An h-adaptive method in the generalized finite differences[J]. Computer methods in applied mechanics and engineering, 2003, 192(5–6): 735–59.

[127] Ureña F, Salete E, Benito J J, et al. Solving third- and fourth-order partial differential equations using GFDM: application to solve problems of plates[J]. International Journal of Computer Mathematics, 2012, 89(3): 366-376.

[128] Wang Y, Gu Y, Fan C M, et al. Domain-decomposition generalized finite difference method for stress analysis in multi-layered elastic materials[J], Engineering Analysis with Boundary Elements. 2018, 94:94–102.

[129] Li P W, Fan C M. Generalized finite difference method for two-dimensional shallow water equations[J], Engineering Analysis with Boundary Elements. 2017, 80: 58–71.

[130] Fu Z J, Xie Z Y, Ji S Y, et al. Meshless generalized finite difference method for water wave interactions with multiple-bottom-seated-cylinder-array structures[J], Ocean Engineering, 2020, 195:106736.

[131] Rao X. An upwind generalized finite difference method (GFDM) for meshless analysis of heat and mass transfer in porous media[J]. Computational Particle Mechanics, 2023, 10: 533–554.

[132] Benito J J, Ure˜na F, Gavete L, et al. Implementations with generalized finite differences of the displacements and velocity-stress formulations of seismic wave propagation problem[J], Applied Mathematical Modelling, 2017, 52: 1–14.

[133] Liu Y, Rao X, Zhao H, et al. Generalized finite

difference method based meshless analysis for coupled two-phase porous flow and geomechanics[J]. Engineering Analysis with Boundary Elements, 2023, 146: 184-203.

[134] Xia H, Gu Y, Short communication: The generalized finite difference method for electroelastic analysis of 2D piezoelectric structures[J], Engineering Analysis with Boundary Elements. 2021, 124: 82–86.

[135] Gu Y, Qu W, Chen W, et al. The generalized finite difference method for long-time dynamic modeling of three-dimensional coupled thermoelasticity problems[J], Journal of Computational Physics, 2019, 384: 42–59.

[136] Rao X, Liu Y, Zhao H. An upwind generalized finite difference method for meshless solution of two-phase porous flow equations[J]. Engineering Analysis with Boundary Elements, 2022, 137: 105-118.

[137] Gu Y, Wang L, Chen W, et al. Application of the meshless generalized finite difference method to inverse heat source problems[J]. International Journal of Heat and Mass Transfer, 2017, 108(A): 721-729.

[138] Qu W, He H, A spatial–temporal GFDM with an additional condition for transient heat conduction analysis of FGMs[J], Applied Mathematics Letters, 110 (2020) 106579.

[139] Zhan W, Rao X, Zhao H, et al. Generalized finite difference method (GFDM) based analysis for subsurface flow problems in anisotropic formation[J]. Engineering Analysis with Boundary Elements, 2022, 140: 48-58.

[140] Fan C M, Li P W. Generalized finite difference method for solving two-dimensional Burgers' equations[J], Procedia Engineering, 2014, 79 :55–60.

[141] Fu Z J, Tang Z C, Zhao H T, et al. Numerical solutions of the coupled unsteady nonlinear convection–diffusion equations based on generalized finite difference method[J], Eur. Phys. J. Plus 2019, 134 (6): 272.

[142] Gu Y, Sun HG. A meshless method for solving three-dimensional time fractional diffusion equation with variable-order derivatives[J]. Applied Mathematical Modelling, 2020, 78: 539-549.

[143] Chan H F, Fan C M, Kuo C W. Generalized finite difference method for solving two-dimensional non-linear obstacle problems[J]. Engineering Analysis with Boundary Elements, 2013, 37(9): 1189-1196.

[144] Rao X. An upwind generalized finite difference method (GFDM) for meshless analysis of heat and mass transfer in porous media[J]. Computational Particle Mechanics, 2023, 10(3): 533-554.

[145] Rao X, Zhao H, Liu Y. A novel meshless method based on the virtual construction of node control domains for porous flow problems[J]. Engineering with Computers, 2024, 40(1): 171-211.

[146] Rao X, Zhao H, Liu Y. A meshless numerical modeling method for fractured reservoirs based on extended finite volume method[J]. SPE Journal, 2022, 27(06): 3525-3564.

[147] Qian Zhi, Hu Xinhong, Yang Hongwei, et al. Comprehensive utilization of multiple logging curves for formation division and correlation [J]. Petroleum Instruments, 2008(05): 46-47+52+25.

[148] Wu Hao. Optimized inversion of interwell connectivity based on production logging data [D]. Yangtze University, 2021.

[149] Wang Xiang, Xia Zhujun, Zhang Hongwei, et al. Research on the method of identifying large-scale channels using water injection profile logging data [J]. Well Logging Technology, 2002, 26(2): 162-164.

[150] Meng Fanshun, Sun Tiejun, Zhu Yan, et al. Identification of large-scale channels in sandstone reservoirs using conventional logging data [J]. Periodical of Ocean University of China (Natural Science Edition), 2007, 37(3): 463-468.

[151] Hong Jinxiu, Xu Wei, Tu Guoping. Formation mechanism of large-scale channels in fluvial-deltaic depositional reservoirs and identification method using DLS logging curves [J]. Petroleum Geology and Oilfield Development in Daqing, 2006, 10(4): 51-53+122.

[152] Zhao Xiaoqing, Pan Baozhi, Zhu Daoping, et al. Evaluation of large-scale channels in reservoirs

using carbon-oxygen ratio (C/O) spectral logging data [J]. Well Logging Technology, 2009, 33(2): 4.

[153] Shang Hailan, Wei Yanmei, Li Zhigang, et al. Application of pulsed neutron oxygen activation technology to guide the treatment of large-scale channels in Jing-11 fault block [J]. Well Testing, 2009, 3(5): 66-67+78.

[154] Kaufman R L, Ahmed A S, Hempkins W B. A new technique for the analysis of comingled oils and its application to production allocation calculations[J]. indonesian petroleum association, 1987.

[155] Wen Zhigang, Zhu Dan, Li Yuquan, et al. Application of chromatography fingerprinting technology in the study of reservoir connectivity in Block 6 of Gudong Oilfield [J]. Petroleum Exploration and Development, 2004, 31(1): 82-83.

[156] Li Ning. Research on the interpretation and application model of multi-well reservoir dynamic testing data [D]. Xi'an Shiyou University, 2012.

[157] Ding Yao. Application research on interwell dynamic connectivity inversion method for high water-cut reservoirs [D]. China University of Petroleum (Beijing), 2017.

[158] Li Yuegang, Xu Wen, Xiao Feng, et al. Optimization of development well patterns based on dynamic characteristics [J]. Natural Gas Industry, 2014, 34(11): 56-61.

[159] Wan Xinde, Wu Yi. Application of pulse testing in oilfield development [J]. Special Oil & Gas Reservoirs, 2006, 13(3): 66-69.

[160] Su Qunying, XiongNian, Zhang Jing. Application of pulse testing to analyze interwell connectivity in polymer-flooded reservoirs [J]. Liaoning Chemical Industry, 2012, 41(09): 951-952.

[161] Liu Zhenyu, Zeng Zhaoying, Zhai Yunfang, et al. Study on connectivity of low permeability reservoirs using pulse testing method [J]. Acta Petrolei Sinica, 2003, 24(1): 73-77.

[162] Tiab D, Dinh A V. Inferring interwell connectivity from well bottomhole-pressure fluctuations in waterfloods[J]. SPE Reservoir Evaluation & Engineering, 2008, 11(05):874-881.

[163] Liao Hongwei, Wang Chen, Zuo Dairong. Determination of interwell connectivity using unsteady-state well testing [J]. Petroleum Exploration and Development, 2002, 29(4): 87-89.

[164] Lin Jia'en, Wang Yaozu, Yang Jinghai. Qualitative analysis of injection-production balance and interwell connectivity using unsteady-state pressure testing data [J]. Well Testing, 1997, 6(2): 4-9.

[165] Shi Yougang, Zeng Qinghui, Zhou Xiaojun. Theoretical interpretation model of well testing for large-scale channels [J]. Oil Drilling & Production Technology, 2003, 15(3): 48-50+84.

[166] Muskat M. A note on a problem in potential theory[J]. Journal of Applied Physics, 1937, 8(6): 434-440.

[167] Fay J A, & Pratts S A. The Flow of Permeable Media[J]. In Journal of Applied Physics, 1951, 22(5), 655-658.

[168] Martin J C, Woo P T, Wegner R E. Failure of Stream Tube Methods to Predict Waterflood Performance of an Isolated Inverted Five-Spot at Favorable Mobility Ratios[J]. JPT, 1973, 25: 151-153.

[169] Higgins R V, Leighton A J. Computer Prediction of Water Drive of Oil and Gas Mixtures Through Irregularly Bounded Porous Media-Three-Phase Flow[J]. JPT, 1962, 14: 1048-1054.

[170] Martin J C, Wegner R T. Numerical Solution of Multiphase, Two-Dimensional Incompressible Flow Using Streamtube Relationships[J]. Society of Petroleum Engineers Journal, 1979, 19: 313-323.

[171] Lake L W, Johnston J R, Stegemeier G L. Simulation and Performance Prediction of a Large-Surfactant/Polymer Project[J]. Society of Petroleum Engineers Journal, 1981, 21: 731-739.

[172] Pollock D W. Semianalytical Computation of Path Lines for Finite-Difference Models[J]. Ground Water, 1988, 26(6): 743-750.

[173] Datta-Gupta A, King M J. A Semianalytic Approach to Tracer Flow Modeling in Heterogeneous Permeable Media[J]. SPE Formation Evaluation, 1995, 10: 40-48.

[174] Peddibhotla S, Cubillos H, Datta-Gupta A, et al. Rapid simulation of multiphase flow through fine-scale geostatistical realizations using a new, 3-D, streamline model: a field example[C]//SPE Petroleum Computer Conference. SPE, 1996: SPE-36008-MS.

[175] Zhao Hui, Kang Zhijiang, Zhang Yun, et al. A connectivity method for characterizing interwell formation parameters and oil-water performance [J]. Acta Petrolei Sinica, 2014, 35(5): 922-927.

[176] Zhao H, Kang Z, Zhang X, et al. INSIM: A data-driven model for history matching and prediction for waterflooding monitoring and management with a field application[C] SPE reservoir simulation symposium, 2015, SPE-173213-MS.

[177] Zhao H, Kang Z, Zhang X, et al. A physics-based data-driven numerical model for reservoir history matching and prediction with a field application[J]. SPE Journal, 2016, 21(06): 2175-2194.

[178] Zhao H, Li Y, Cui S, et al. History matching and production optimization of water flooding based on a data-driven interwell numerical simulation model[J]. Journal of Natural Gas Science and Engineering, 2016, 31: 48-66.

[179] Zhao H, Kang Z, Sun H, et al. An interwell connectivity inversion model for waterflooded multilayer reservoirs[J]. Petroleum Exploration and Development, 2016, 43(1): 106-114.

[180] Xie X, Zhao H, Kang X, et al. Prediction method of produced polymer concentration based on interwell connectivity[J]. Petroleum Exploration and Development, 2017, 44(2): 286-293.

[181] Guo Z, Reynolds A C, Zhao H. A physics-based data-driven model for history matching, prediction, and characterization of waterflooding performance[J]. SPE Journal, 2018, 23(02): 367-395.

[182] Guo Z, Reynolds A C, Zhao H. Waterflooding optimization with the INSIM-FT data-driven model[J]. Computational Geosciences, 2018, 22: 745-761.

[183] Zhao H, Xu L, Guo Z, et al. A new and fast waterflooding optimization workflow based on INSIM-derived injection efficiency with a field application[J]. Journal of Petroleum Science and Engineering, 2019, 179: 1186-1200.

[184] Guo Z, Reynolds A C. INSIM-FT in three-dimensions with gravity[J]. Journal of Computational Physics, 2019, 380: 143-169.

[185] Zhao H, Xu L, Guo Z, et al. Flow-path tracking strategy in a data-driven interwell numerical simulation model for waterflooding history matching and performance prediction with infill wells[J]. SPE Journal, 2020, 25(02): 1007-1025.

[186] Liu W, Zhao H, Sheng G, et al. A rapid waterflooding optimization method based on INSIM-FPT data-driven model and its application to three-dimensional reservoirs[J]. Fuel, 2021, 292: 120219.

[187] Zhao H, Liu W, Rao X, et al. INSIM-FPT-3D: A Data-driven Model for History Matching, Water-breakthrough Prediction and Well-connectivity Characterization in Three-dimensional Reservoirs[C]. SPE Reservoir Simulation Conference, Galveston, Texas, USA, 4–6 October 2021, SPE-203931-MS.

[188] Liu W, Zhao H, Zhong X, et al. A Novel Data-Driven Model for Dynamic Prediction and Optimization of Profile Control in Multilayer Reservoirs[J]. Geofluids, 2021, 3272860.